Physics:

what you need to know

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Directions

This study guide provides explanations for various physics topics.

Homeschool students can use CK12.org Physics Textbook

Your course may not cover all of the topics in this guide, and it may include information not discussed here. Your class notes are the best guide on what to study for exams. Many students taking AP Physics will also take calculus, so some relevant calculus explanations are included in this guide. These sections are clearly marked, and you should ignore them if you have not taken calculus. If you do know calculus, it is worth your time to read these explanations even if they are not included in your course. Homeschool students can afford to spend extra time to understand basic concepts and do some simple experiments.

EQUATION SUMMARY

Relevant equations are provided here for easy reference. You may not need all of them depending on the level of your course.

Velocity: $v = \frac{d}{t}$ $v = v_0 + at$ Initial velocity may be shown as v_0 or v_i in your textbook.

Acceleration: $a = \frac{v - v_0}{t}$

Displacement ($d = x - x_0 = \Delta x$):

$d = vt$ (constant velocity)

$d = v_{ave}t = \frac{v_0 + v}{2}t$ (constant acceleration)

$d = v_0 t + \frac{1}{2}at^2$

$d = \frac{v^2 - v_0^2}{2a}$

Force: $F = m\,a$

Force of Gravity = $\frac{G \cdot M_1 M_2}{\text{distance}^2}$

Kinetic Energy: $KE = ½\, mv^2$

Work: $W = F\,d$

Work done by gravity: $W = F\,h = m\,g\,h$

Work = change in kinetic energy: $W = ½\, m\, v^2 - ½\, m\, v_0^2$

Find the speed of a falling object: $gh = ½\, v^2 - ½\, v_0^2$

Friction = normal force · coefficient of friction

Force = $\frac{\Delta \text{ momentum}}{\Delta \text{ time}}$

Impulse = Δ momentum = $F\Delta t$

Position of the Center of Mass: $x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

Velocity of the Center of Mass: $v = \frac{m_1 v_1 + m_2 v}{m_1 + m_2}$

Circular Motion:

$a = \frac{v^2}{r}$

Centripetal force: $F_c = m\frac{v^2}{r}$

Arc length: $S = r\theta$

Angular Velocity: $\omega = \frac{\Delta\theta}{\Delta t}$

Linear Velocity: $v = \omega r$

Angular Acceleration: $\alpha = \frac{\omega}{t}$ and $a = \alpha\, r$

Torque: $\tau = F\, r$

Work = $Fd = \tau\theta$

$\tau = I\,\alpha$ where $I = mr^2$

Angular Momentum: $L = I\,\omega$

Rotational KE = $\frac{1}{2} I\,\omega^2$

Banked Roads: $v^2 = g\, r \tan\theta$

Machines:

Mechanical advantage = $\frac{\text{Output Force}}{\text{Input Force}}$

Efficiency = $\frac{\text{Output Work}}{\text{Input Work}}$ x 100%

Simple Harmonic Motion:

Hooke's Law: $F = -kx$

$PE_{Spring} = \frac{1}{2} kx^2$

Period of a Spring: $T = 2\pi\sqrt{\frac{m}{k}}$

Period of a Pendulum: $T = 2\pi\sqrt{\frac{L}{g}}$

Electricity:

$F = k\frac{q_1 q_2}{r^2}$

$I = \frac{V}{R}$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots.$

$R = \frac{\rho L}{A}$

Electrical Power: $P = I\,V$

SIGNIFICANT FIGURES

Zeros that come after the decimal point mean a lot more to scientists than they do to mathematicians. When a physicist says that an object weighs 38.0 grams, he doesn't just mean 38 grams. That last zero is the result of a careful measurement, and it is just as important as the other digits, so don't lose it! When you do calculations with measurements, the final answer should be just as precise as the original measurements. To accomplish that we have to understand which part of a number is "significant".

You have already learned to use conversion factors, so let's take the measurement 38.0 grams and convert it to kilograms. To do that, we have to know (or look up) that there are 1000 grams in a kilogram. We multiply 38.0 grams $\cdot \frac{1\text{ kg}}{1000\text{grams}}$. The unit "grams" cancels out and we are left with $\frac{38.0\text{ kg}}{1000}$ = 0.0380 kilograms. There are now more digits in our measured number, but converting the measurement to a different unit doesn't make it more precise. The two zeros in front of the 3 are really just "placeholder zeros" that will not be there if we switch back to grams. We say that 38.0 has 3 significant digits, and so does 0.0380.

Zeroes that are somewhere in the middle of a number are obviously significant, so 705 has 3 significant digits. The number 12001 has 5 significant digits. Unfortunately however, when there are zeros at the end of a number we sometimes have to guess what kind of measurement was actually made. For example, you may hear people say that the speed of light is 300,000 km per second. That is an amazingly convenient figure. Does it mean that the speed was measured exactly to the nearest kilometer, and found to be exactly 300,000 km/sec, or was it rounded off? Actually it was, and there is a far more precise figure available. To the nearest kilometer, the speed of light is 299,792 km/sec. That gives the speed of light correct to 6 significant figures. Measuring to the nearest meter gives 299 792 458 meter/sec. This last measurement has 9 significant figures. If you want to show that you have measured something to be precisely 2000 kilometers, you can indicate that by writing 2000. kilometers. The decimal point indicates that the last zero is significant, so your number has 4 significant figures. Some textbooks will place a line under or over ambiguous zeroes to indicate that they are significant. The best way to do things though is to use scientific notation. "Exactly 2000 kilometers" can be written as 2.000×10^3 km. This clearly shows that the measurement was not rounded off and all those zeroes matter. Ask your teacher what to do when a figure like 2000 km is not given in scientific notation on a test.

Scientific notation also helps by getting rid of those potentially confusing zeroes at the beginning of numbers. To write 0.0380 in scientific notation, consider that 0.0380 is the same as $3.80 \times \frac{1}{100}$. Because $\frac{1}{100}$ can be written as 10^{-2}, the actual notation used is 3.80×10^{-2}. Now it is clear that 0.0380 has 3 significant figures. The zeros at the beginning of the number are not involved in showing how precise the measurement was.

When you start doing calculations, you may have a problem if one value was measured much more precisely than another. Suppose a developer wants to buy two adjacent properties, belonging to neighbors Jack and Sam. Jack is a very organized person who has kept all documents about his property. He reports that the area of his land is 8.35 acres. Sam can't find his papers anywhere, but he remembers that he has about 10 acres of land. It would not make much sense to say that buying both properties would give the developer exactly 18.35 acres of land. Sam's figure is only correct to the nearest whole acre. The best we can do when adding or subtracting is to use the lowest available correct place value. We report the total to the nearest whole acre, or 18 acres. If Sam had said that he has 10.1 acres of land, we would have our information correct to the nearest tenth of an acre. In that case we would report the total as 18.5 acres (10.1 + 8.35 = 18.45, which is 18.5 when rounded to the nearest tenth).

When you are multiplying or dividing, keep as many significant figures as practical during your calculations to avoid rounding errors. However, the final result of multiplication and division can have only as many significant figures as the number with the least amount of significant figures that you used as a basis for your calculations. For example: 16.54 ÷ 19.00 x 2.7 = 2.4 Both 16.54 and 19.00 have four significant figures, but 2.7 only has two. The final result must be rounded off to two significant figures.

MEASURING MOTION

We determine speed by measuring how long it takes for something or someone to travel a known distance. Then we divide the distance by the time.

speed = $\mathbf{\frac{distance}{time}}$

It makes sense that speed is measured in units like miles per hour, or meters per second.

Just like you can rearrange $3 = \frac{6}{2}$ to: $2 = \frac{6}{3}$ or 6 = 2 x 3, you can rearrange the equation for speed:

time = $\mathbf{\frac{distance}{speed}}$ or **distance = speed x time**

When we start analyzing motion more precisely, we also want to consider the direction of the movement. For example, you can throw a ball straight up. The ball is moving fast initially, but at some point it stops and reverses direction to head back down. To describe this motion more precisely, we can think of “up” as positive, and “down” as negative. In fact, we might want to set up an imaginary vertical number line, kind of like a y-axis, with the 0 point at the spot where the ball leaves your hand. The speed of the ball as it goes up is not constant. Suppose the ball reverses direction after traveling 36 feet, exactly 1.5 seconds after it leaves your hand. We can find the **average speed** on the way up by taking the distance and dividing it by the time:

Average speed = $\frac{36 \text{ feet}}{1.5 \text{ seconds}}$ = 24 feet/sec.

Then the ball starts to fall. You catch it at the same point where it originally left your hand exactly 1.5 seconds from the time it started to fall. Again you calculate the average speed to be 24 feet/sec, but now that speed is in the opposite direction. We can account for this by considering the ball’s **velocity**, which is speed with a direction. When the ball is going up, the velocity is positive, and when it is coming down the velocity is negative. The distance in this case is also in two different directions, up and down. It makes sense to look at the change in position of the ball along a numbered axis (its **displacement**), and considering its velocity instead of its speed.

displacement = final position – initial position

velocity = $\mathbf{\dfrac{displacement}{time}}$

At first the ball moves from position 0 to position 36. Its displacement is its final position minus its initial position: 36 – 0 = 36 feet. The velocity of the ball is $\dfrac{\text{displacement}}{\text{time}} = \dfrac{36 \text{ feet}}{1.5 \text{ seconds}} =$ 24 feet/sec.

Next, the ball moves from position 36 to position 0. Its displacement is 0 – 36 = -36 feet. The velocity of the ball is $\dfrac{-36 \text{ feet}}{1.5 \text{ seconds}}$ = -24 feet/sec.

Because the velocity is +24 feet/sec for 1.5 seconds, and then -24 feet/sec for 1.5 seconds, we expect both the total displacement and the total average velocity to be 0:

Displacement = final position – initial position = 0 – 0 = 0.

Average velocity = $\dfrac{\text{displacement}}{\text{time}} = \dfrac{0 \text{ feet}}{3 \text{ seconds}}$ = 0 feet/sec

Notice that while the average velocity for the entire movement of the ball is zero, the average speed is not. Speed is *distance* over time, and the ball travels a total distance of 36 feet + 36 feet = 72 feet. The average speed is $\dfrac{72 \text{ feet}}{3 \text{ seconds}}$ = 24 feet/sec.

When the velocity is not constant, we can measure how much it is increasing or decreasing. The change in velocity over time is the acceleration:

Acceleration = $\mathbf{\dfrac{change\ in\ velocity}{time}}$

Let's call the acceleration a, the time t, the final velocity v, and the initial velocity v_0 (the velocity at time 0, which is the time when you begin measuring).

a = $\mathbf{\dfrac{v - v_0}{t}}$

You may also see this formula written as **a =** $\mathbf{\dfrac{v_f - v_i}{t}}$, where v_f means the final velocity, and v_i means the initial velocity.

Like velocity, acceleration has a direction. If an object is accelerating in the same direction as it is already moving, it will speed up. If the acceleration is in the opposite direction the object will slow down. In real life we call the slowing down deceleration, but physics doesn't normally use that term. You might think that negative acceleration would mean the object is slowing down, but things don't work that way. If the velocity is negative, say -2m/sec, and then gets more negative, like maybe to -5m/sec over a period of 1 second, the acceleration will be negative even though the object is speeding up. Just fill in the formula:

$a = \frac{v - v_0}{t} = \frac{-5 - -2}{1} = -3$

Working with Acceleration

Acceleration is the change in velocity, divided by the elapsed time. Velocity has standard units of meters per second. When you divide that by the time, in seconds, you get meters per second per second:

$$\frac{\text{meters}}{\text{second}} \div \text{second} = \frac{\text{meters}}{\text{second}} \times \frac{1}{\text{second}} = \frac{\text{meters}}{\text{second}^2}$$

Although we usually say meters per second squared, it actually helps to say meters per second per second when you are trying to understand what is happening during acceleration.

Suppose an object starts at rest, and then accelerates at a rate of 5m/sec^2. How fast is it moving after 4 seconds? Well, the speed is changing at a rate of 5 meters/second every second. So, after 1 second the speed is 5m/sec, after 2 seconds it is 10m/sec, after 3 seconds it is 15m/sec, and after 4 seconds the speed is 20 m/sec.

You can see that this answer is correct by using the formula for acceleration:

$$a = \frac{v - v_0}{t}$$

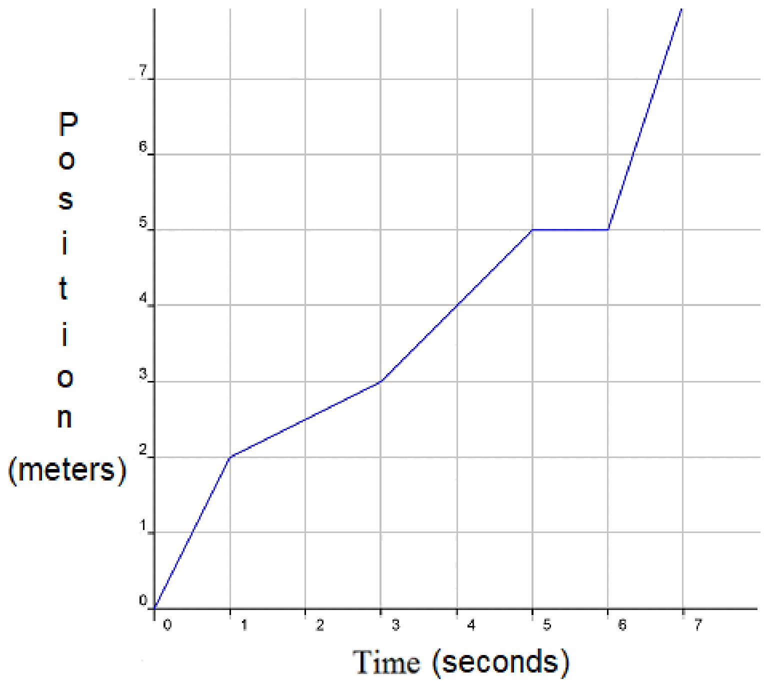
$$a = \frac{20\text{ m/sec} - 0\text{ m/sec}}{4\text{ sec}}$$

$$a = 5\text{ m/sec}^2$$

Motion Graphs

Graphs are a good way to look at motion. The two kinds of graphs you are likely to see are "Position versus Time" and Velocity versus Time". Make sure you know which one you're looking at, because that's a big difference!

In the graph below you can see that the *position* is changing during some of the time intervals. However, between t = 5 seconds and t = 6 seconds the position does not change. The object is stationary during this time.

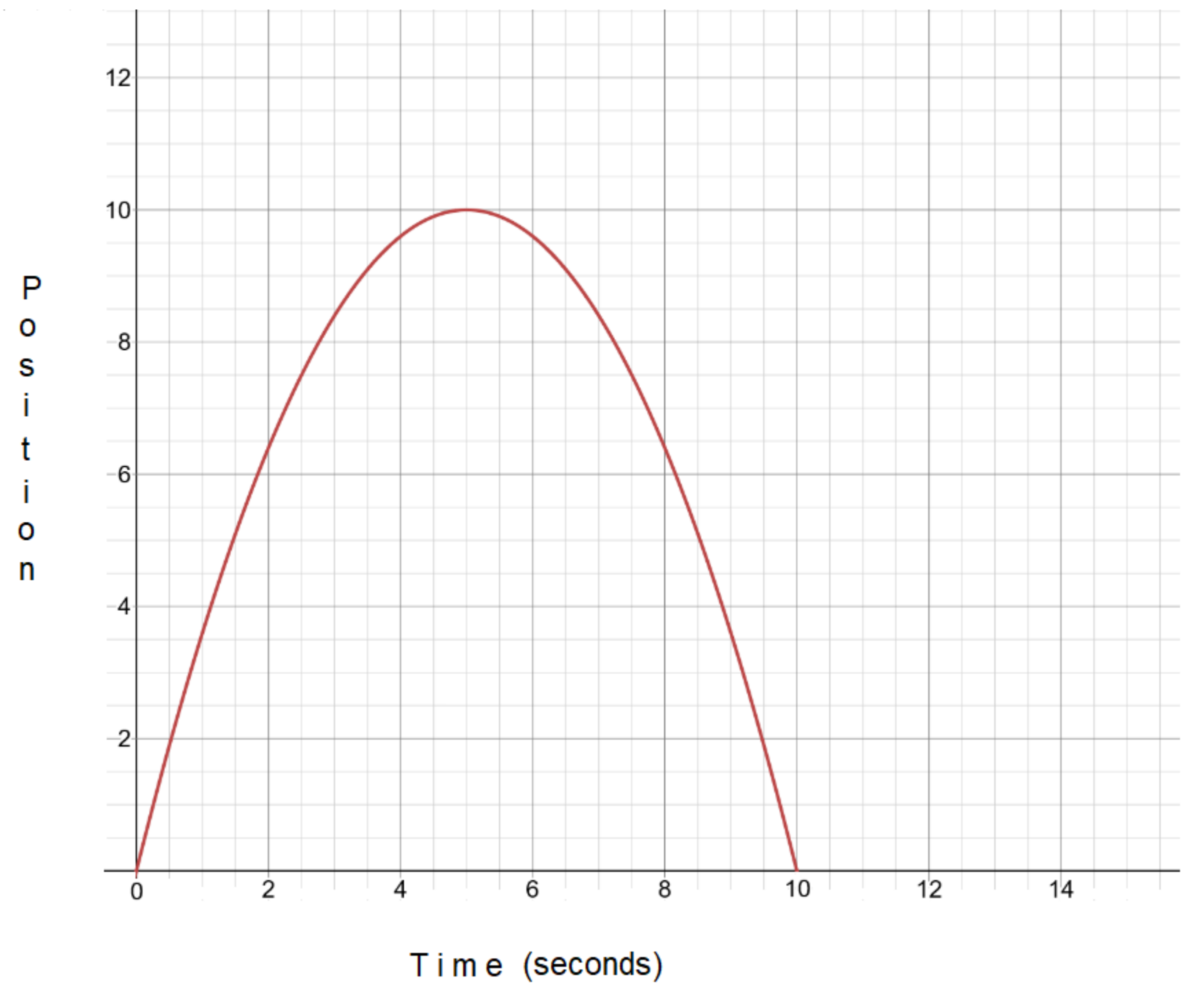


During the first second there is a fairly rapid change in position, and then the position changes by only 1 meter between t = 1 and t = 3. The change in position over time (the **rate of change**) is the velocity, and it is not difficult to calculate the velocity from this graph. In the first second the velocity is 2 m/sec. For straight lines on a position vs time graph, the velocity is the slope of the line. If the slope is zero the velocity is zero. A negative velocity is indicated by a negative slope. Look at the graph carefully to make sure you can tell during which interval the velocity is the greatest.

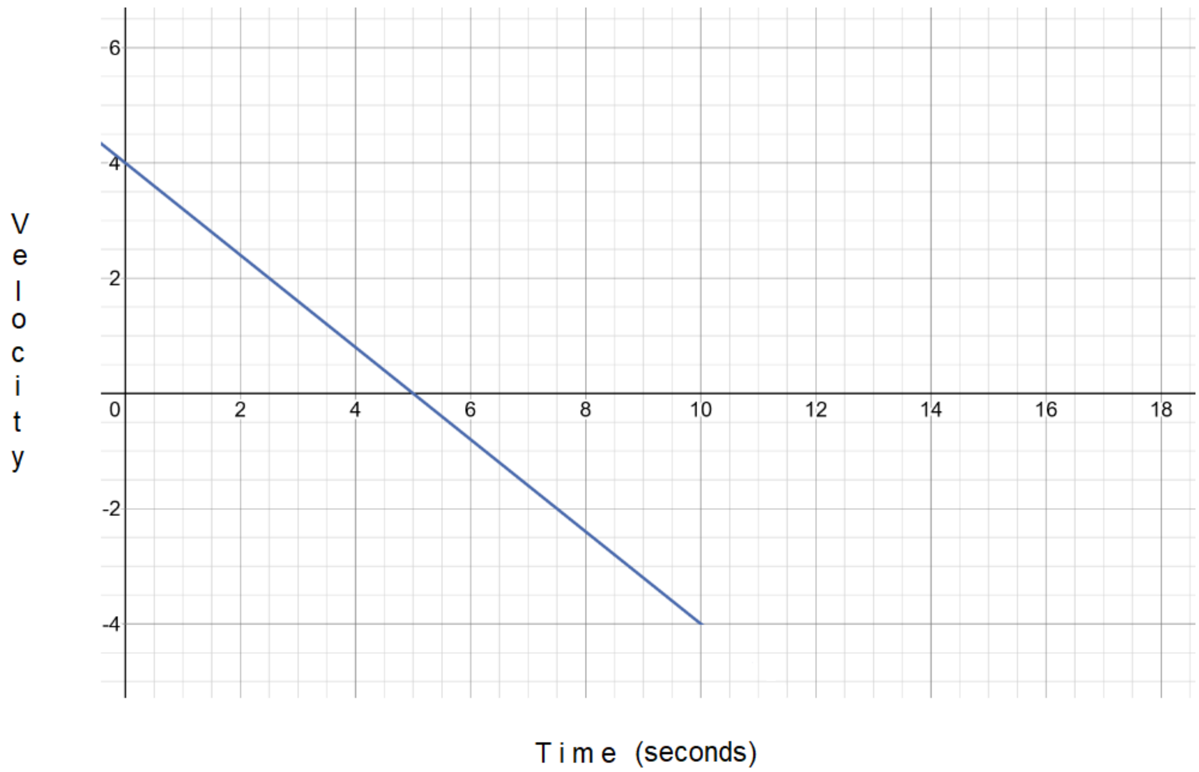
The **average rate of change** is the average velocity in this case. To find the average velocity, determine the change in position and divide by the elapsed time. For example, between t = 1 and t = 6, the change in position is 5 meters – 2 meters = 3 meters. The elapsed time is 6 seconds – 1 second = 5 seconds. The average velocity is 3/5 or 0.6 meters per second.

If the change in position is shown as a curve rather than a straight line, you still find the average velocity in the same way: divide the displacement by the elapsed time. All that matters is how far the object moved in the given time, not how it got there.

For a *velocity* versus time graph, a horizontal line means that the velocity is not changing. Unless that horizontal line is at zero, the object is still moving! The rate of change of the velocity is the acceleration. You can use the slope to find the acceleration, or calculate the average acceleration by finding the velocity at the end of the interval and at the start of the interval. Divide the change by the elapsed time.



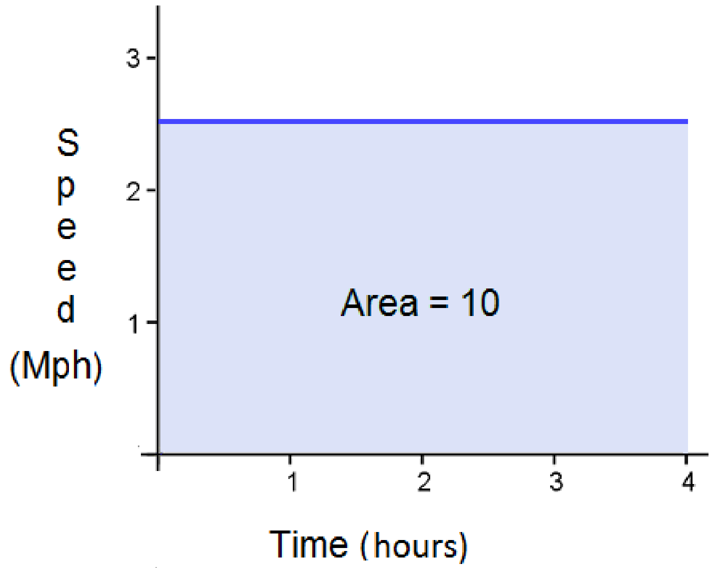
This graph shows an object going straight up and then coming straight down. The fact that we are stretching the motion out over time in the graph makes it look like a parabola. Notice that at first the object is moving fast, and then it slows down and reverses direction. However, a graph of the velocity (shown below) shows that the velocity is decreasing steadily over the same time interval. That may seem strange, but if you compare the graphs carefully you can see what is happening. The velocity has a large positive value at first. Then it decreases to zero at t = 5 seconds, and after that it becomes more and more negative, which indicates that the object is speeding up before it hits the ground.



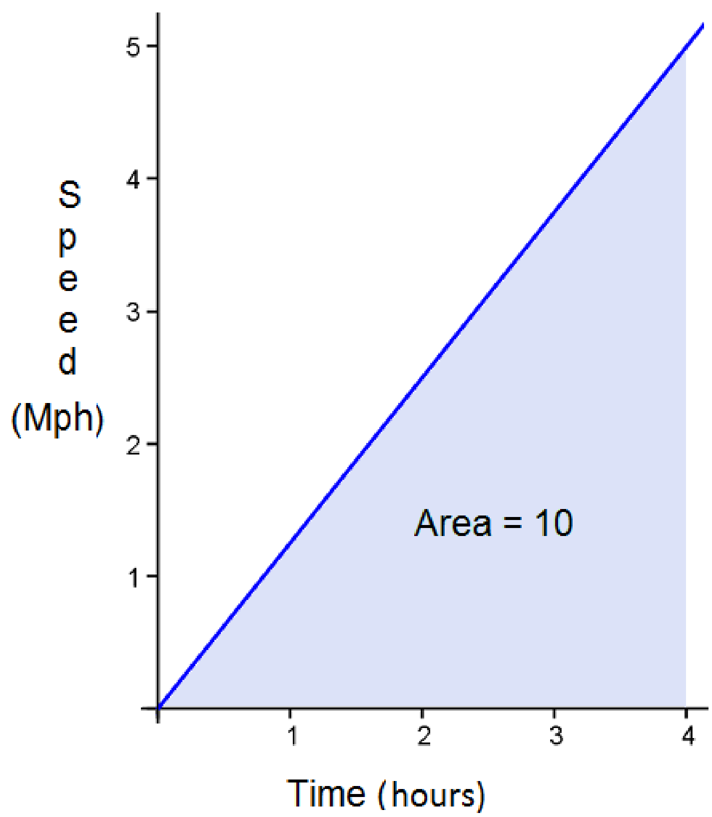
The acceleration, which is the change in the velocity over time, is negative as you can see from the negative slope of the velocity graph. The acceleration is constant. In this case the acceleration was caused by gravity, which was the only force that acted on the object after it was launched from the ground.

Notice that a negative acceleration can cause an object to either slow down or speed up, depending on the direction in which it is moving.

Consider the following speed versus time graph:

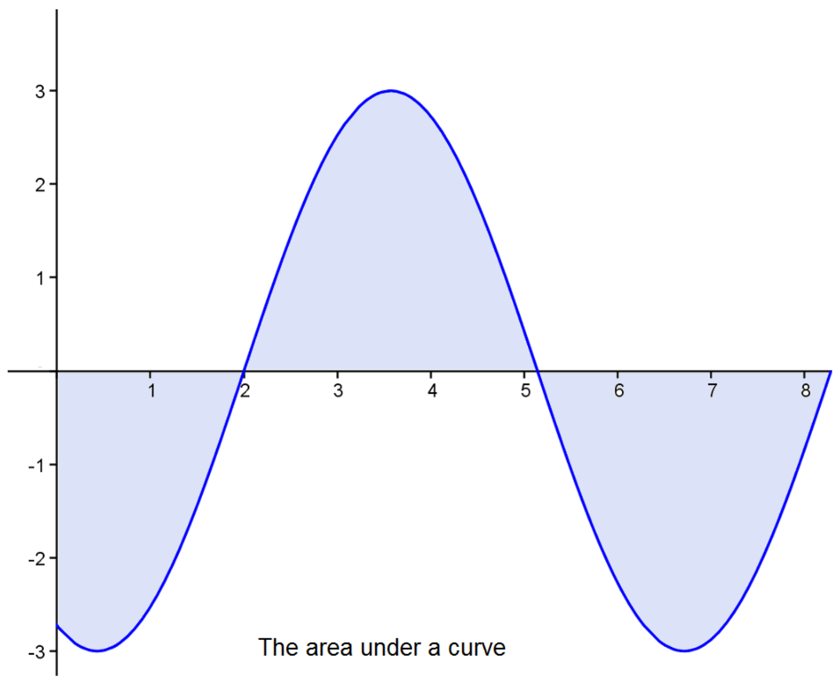


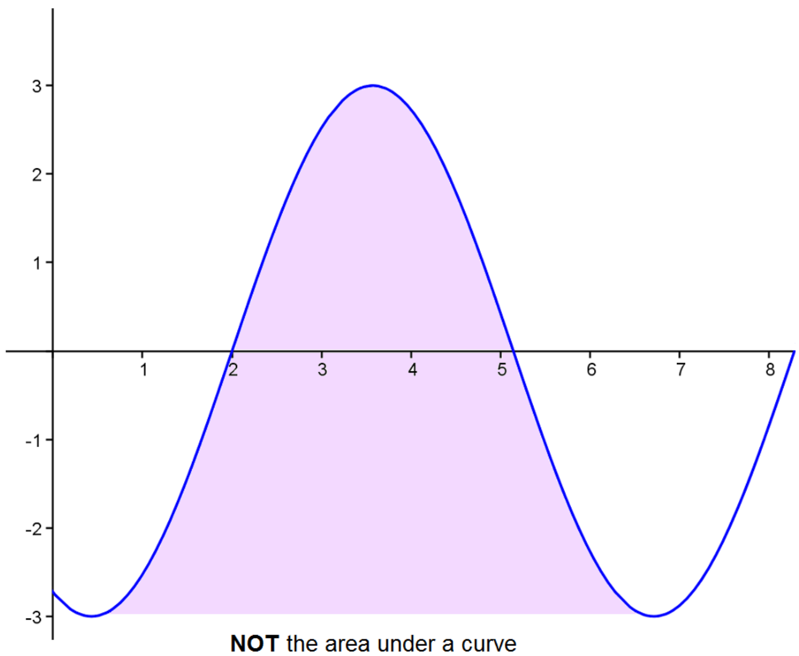
If you walked for 4 hours, at a leisurely speed of 2.5 miles per hour, you would cover a distance of 10 miles. The area under the curve represents the distance you travel (speed x time). An *average* speed of 2.5 miles per hour would allow you to cover the same distance:



Regardless of the shape of a speed versus time graph, the area underneath it represents the distance traveled.

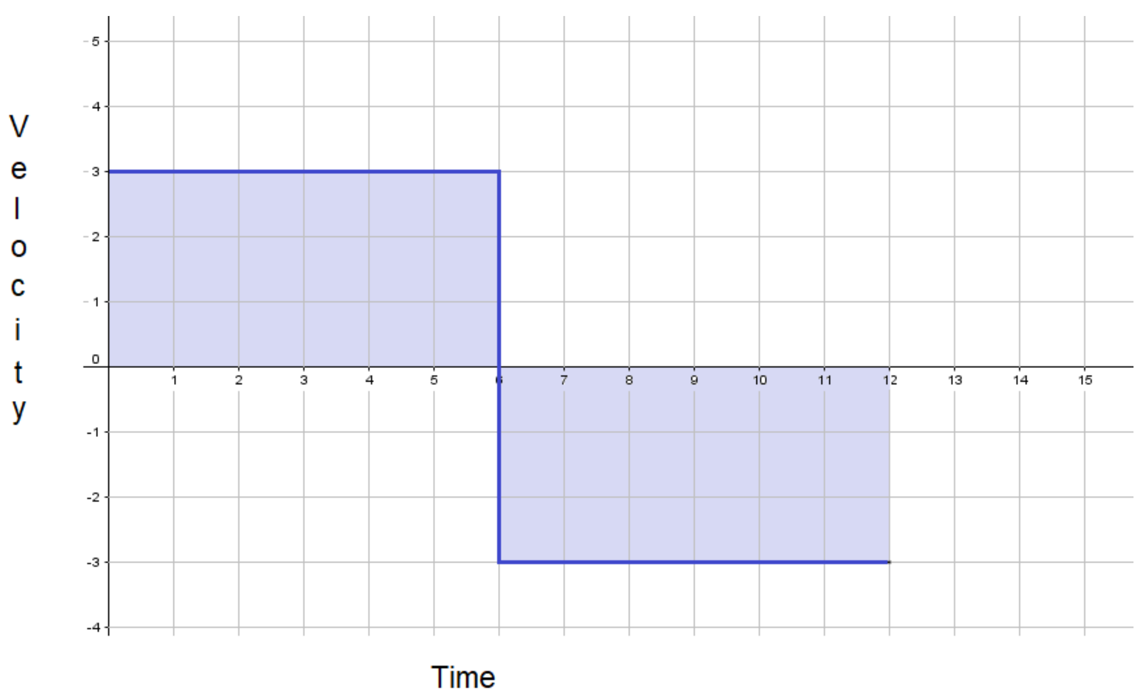
Unlike speed, velocity may be negative. For a velocity versus time graph, the area under the graph means the area between the line and the x-axis:





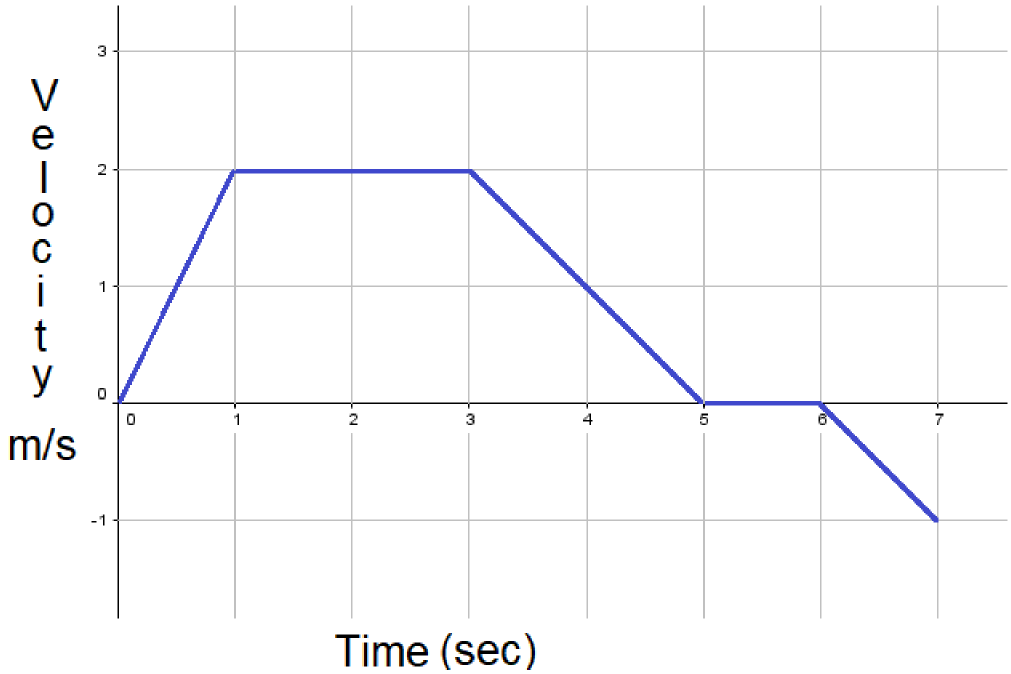
NOT the area under a curve

Any part of the area that lies below the x-axis is considered negative. The area under a velocity versus time graph represents the displacement rather than the distance traveled. In the graph below, the velocity changes from positive to negative at t = 6. The area under the graph is 18 + - 18 = 0. The displacement is zero units, although the distance traveled is 36 units.



Example

The graph below shows the velocity of an object traveling along a horizontal track. If the initial position is at zero meters, what is the final position of this object after 7 seconds? During which intervals, if any, is the object standing still?



Displacement: 1 m + 4 m + 2 m + 0 m + -0.5 m = 6.5 m. Zero velocity between t = 5 sec and t = 6 seconds. Note that between 1 and 3 seconds the object is moving at 2 m/sec.

Important Formulas

Velocity is displacement over time, $v = \frac{d}{t}$, so displacement is equal to velocity x time:

$d = v\,t$

But what if the speed is not constant? Well, if it is just changing at a nice steady rate (as it is in most physics problems), we can use the average speed. To get the average of two numbers you add them and divide by 2. If you call the initial speed v_0 and the final speed v, then the average speed is

$v_{ave} = \frac{v_0 + v}{2}$

That gives us a better formula for displacement:

$\mathbf{d = v_{ave}t}$

$\mathbf{d} = \left(\frac{\mathbf{v_0 + v}}{\mathbf{2}}\right)\mathrm{t}$ or $\mathbf{d = ½(v_0 + v)t}$

And by better I mean more accurate, not less complicated.

Acceleration is the change in velocity divided by the time. If some object starts at rest, and then accelerates, $\mathbf{a} = \frac{\mathbf{v - 0}}{\mathbf{t}}$. The velocity is given by $\mathbf{v = a\,t}$. Your physics problems will normally involve constant acceleration, and your formulas will be based on the assumption that the acceleration remains constant.

When something is already moving, and then it accelerates, the final speed is the initial speed, plus the additional speed caused by the acceleration:

$\mathbf{v = v_0 + at}$

Now that we have a good expression for the final velocity v, we can put that into the formula for displacement:

$d = ½(v_0 + v)t$

$d = ½(v_0 + v_0 + at)t$

That multiplies out to $d = ½\,(2v_0t + at^2)$, so we can say that

$\mathbf{d = v_0t + ½\,at^2}$

If you know the final velocity, the acceleration and the time, but you don't have v_0, the initial velocity, you can find what you need by using the formula $v = v_0 + at$ that we saw earlier: $v_0 = v – at$. Once you know v_0, just plug it into $\mathbf{d = v_0t + ½\,at^2}$.

Alternatively, if you like having extra formulas to memorize, you can substitute for v_0 in the original formula for displacement:

$d = ½(v_0 + v)t$

$d = ½\,(v – at + v)t$

Simplify that to get a new formula:

$d = ½\,(2vt – at^2)$

$\mathbf{d = vt – ½at^2}$

The basic equations of motion, **$v = v_0 + at$** and **$d = v_0t + ½ at^2$**, can help you solve most textbook problems. These equations can also be rearranged to get a useful relationship between displacement, constant acceleration, and velocity that does not involve time.

$v = v_0 + at \rightarrow at = v - v_0 \rightarrow t = \frac{v - v_0}{a}$

You could substitute that into $d = v_0t + \frac{1}{2}at^2$, but that is a bit time-consuming because of the square. Remember that $v_{ave} = \frac{v + v_0}{2}$. We can use the average speed because the acceleration will be constant. We also know that $d = v_{ave}t$. Substitute for v_{ave} and t to get:

$d = v_{ave}t$

$d = \frac{v + v_0}{2} \cdot \frac{v - v_0}{a}$

$d = \frac{v^2 - {v_0}^2}{2a}$ or $v^2 - {v_0}^2 = 2ad$

This formula is commonly used to find the final velocity when the elapsed time is not known. It also allows you to calculate the final position of an object if you know the initial position, the initial and final velocity, and the acceleration. If you think about that, it does make sense that if you know those particular things you should be able to predict where the object will end up. Notice that it makes no difference if v_o is positive or negative. If the object is going to fall down, the acceleration will be negative and everything will work out correctly.

We will see this equation again in a slightly different form when we look at the work done by gravity. There it will look more like **$(h - h_0) \cdot a = \frac{v^2 - {v_0}^2}{2}$**, where h is the height from which the object falls.

FORCE AND NEWTON'S LAWS

Galileo used ramps to slow down the effects of gravity. This way he could study moving objects and time their motion carefully. He quickly realized that the smoothness of the ramps was important to minimize the effect of friction. He created two ramps, and rolled a ball down one ramp and up the other one. The ball reached almost the same height from which it was released, and the smoother the ramps (and the ball), the closer the ball came to reaching that height. Then he made the second ramp less steep, but the ball still almost reached the same height. This led Galileo to conclude that if the second ramp was horizontal the ball would roll forever if no friction was involved.

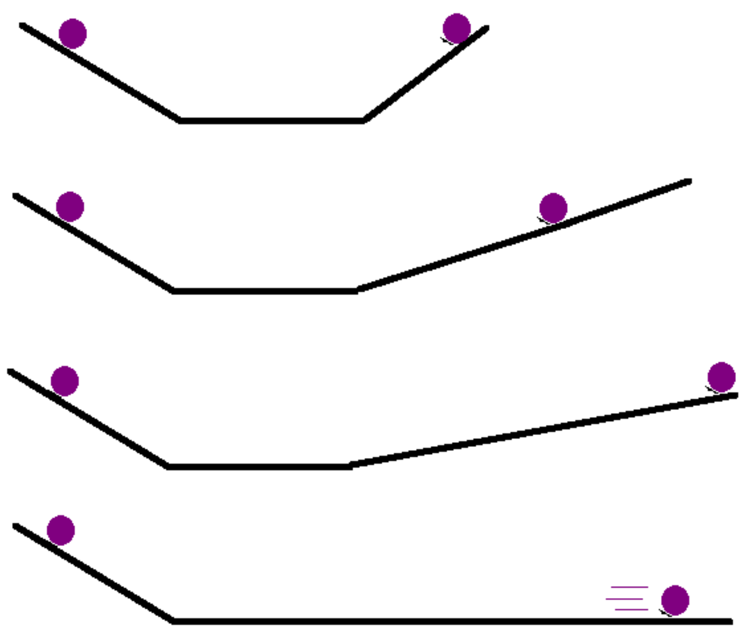


Image from https://pfeifft.wordpress.com/2013/10/28/inertia/

Moving objects only slow down and stop because of friction. Galileo saw that objects have a certain amount of "inertia", which makes them resist changes in their motion. Isaac Newton, who was born the same year that Galileo died, built on these ideas to formulate three laws of motion.

1. Newton's **First Law of Motion**: If something is not moving, it stays that way so long as it is not disturbed. If something is moving, it keeps moving unless it is disturbed. That last part is not so easy to see because on Earth friction slows moving objects. This is also called the Law of Inertia.

2. Newton's **Second Law of Motion**: Things begin to move or stop moving when they are acted on by forces. An object moves faster and faster (it accelerates) if the same force keeps acting on it unopposed by other forces. How fast the object accelerates depends on the size of the force, and also on its mass. **Force = mass · acceleration**

3. The **Third Law of Motion** says that if one object exerts a force on a second object, that second object will also exert a force on the first object. This reactive force has the same magnitude but opposite direction. The force of A on B and the reactive force of B on A form an action-reaction pair.

Newton's first law is often the most difficult one to remember when you are doing problems involving forces. **Whenever a problem states that an object is moving at a constant speed, that means there is no net force acting on it!** For example, if someone lifts a box up *at a constant speed*, there is no net force acting on the box. This means that the force the person is exerting upwards is exactly equal to the force of gravity that is pulling the box down. In the same way, if you slide a box along the floor *at a constant speed*, the force you are applying is exactly equal and opposite to the force of friction between the box and the floor.

What is a **Newton**?

A Newton is a type of apple, named in honor of Sir Isaac Newton who realized that the force that makes an apple fall to the ground is the same force that holds the moon in orbit.

Not to be outdone by some apple grower, scientists have named the unit of force after Newton. This is the unit you use in the equations of Newton's second law of motion. One Newton is the force that will accelerate a 1kg mass at a rate of 1 meter/sec^2:

$$\mathbf{1\ N = 1\ kg \times \frac{1\ m}{sec^2}}$$

Example

An 10 kg object is initially at rest on a frictionless surface. It is pushed forward by a constant force of 5N. How fast is the object moving after 3 seconds?

Answer: According to Newton's second law, Force = mass x acceleration, so
5N = 10 kg x acceleration. From this you can see that the acceleration must be 2 meters/sec^2. That means that the speed is increasing by 2 m/sec every second. After 3 seconds the object's speed will be 6 m/sec.

GRAVITY

Johannes Kepler lived during the same time period as Galileo. He did extensive calculations involving the careful observations that astronomer Tycho Brahe had made of the motion of Mars. Eventually he realized that Mars was moving around the sun in an elliptical orbit rather than a circular one. He formulated three laws that govern the motion of planets.

Kepler's First Law: Planets move in elliptical orbits with the sun at one focus.

Kepler's Second Law: A line drawn from the sun to a planet sweeps out equal areas in equal time intervals. This means that the planets move more slowly when they are further away from the sun, and faster when they are closer.

Kepler's Third Law: The ratio of the square of the orbital periods of any two planets is equal to the ratio of the cubes of their average distances from the sun.

Isaac Newton explained Kepler's laws by showing that the way that the planets move is a natural consequence of the action of gravity, rather than being due to swirls in the celestial ether as proposed by Descartes. You may think that Descartes's theory was ridiculous, but today's view is that the planets move as they do because of distortions in space-time caused by the presence of large masses like the sun. We perceive these distortions as gravity.

Newton's Law of Universal Gravitation says that gravity acts between all objects in the universe. Gravity is everywhere. There is a tiny but measurable gravitational attraction between any two objects. This attractive force depends directly on the mass of both objects. We usually don't notice that because Earth has such a large mass that its attractive force completely overshadows it. When you drop a pencil, it falls because earth attracts it. The pencil also attracts earth, but earth doesn't move measurably because it is so big.

Gravitational force gets weaker as the distance between objects increases. In fact, it gets weaker very quickly, so that increasing the distance 3 times results in the force getting 9 times weaker. Gravity is inversely related to the square of the distance between two objects:

$$\text{Force of Gravity} = \frac{G \cdot M_1 M_2}{\text{distance}^2}$$

Here G is a constant, and M_1 and M_2 are the masses of the two objects. The more total mass there is, the greater the gravitational force. Nearly a century after Isaac Newton first proposed this formula, the value of the gravitational constant G was determined by Lord Henry Cavendish. The modern value for G is $6.67384 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-1}$.

Although the gravitational force decreases rather rapidly with increasing distance, if you stand on a ladder you don't notice that Earth's gravitational pull is any weaker. This is because distance is measured from the center of both objects. Moving further away from Earth's surface does very little to change the distance between you and the center of the Earth. However, normal distances do matter significantly between two objects that are both relatively small. Next time you hug someone, remember that there is in fact a small but measurable gravitational force involved in that hug.

In 1774, Newton's revelations about gravity led to the first experiment that attempted to weigh the Earth. Scientists reasoned that a sufficiently large mountain would attract a mass on a pendulum enough that the pendulum's string would no longer be exactly vertical. The mass of the mountain was carefully determined by measuring its volume and density. This way the mass of the Earth could be compared to the mass of the mountain by determining how much each attracted the pendulum. The experiment was difficult and very time-consuming, and unfortunately the results were not very accurate. Nearly 25 years later, Henry Cavendish constructed an apparatus that twisted due to the attraction between two masses. This allowed for an accurate determination of the force of gravity. Once that was known, the mass of the Earth could be determined by how strongly it attracts a mass with a specific weight.

The pull of gravity on a mass is called its **weight**. A bigger mass has more weight because the pull of gravity on it is stronger. This may make you think that a bigger mass should naturally fall faster than a smaller mass, but we often fail to appreciate the hard work that gravity actually does. Things don't just fall down – they have to be pulled to the ground by the force of gravity. You can use your desk to help you visualize that. Imagine that the right side of your desk is "up", and the left side is "down". The force of gravity would now act horizontally to pull objects from the right side of your desk to the left. Place two objects of different weight on the right side of your desk, for example a glass and a pencil. If you move both the glass and the pencil to the left side of your desk, you can see that it takes more force to get a heavier object to the "bottom" side. However, there is also more force available to pull that heavier object, because the force of gravity on it is stronger. Both these factors involve the mass of the object, so mass cancels out and the glass and the pencil would fall at exactly the same speed if you discount any air resistance.

The fact that objects fall at the same rate regardless of their weight is important, so you should check it out yourself. Legend says that Galileo dropped two cannonballs of different weights from the Leaning Tower of Pisa, and showed that they both hit the ground at the same time. I didn't have cannonballs, so I used a plastic fork and a metal fork of about equal size. You can find your own suitable objects. If you have a large and a small marble, or two balls of different weights, you should roll them down a ramp so the "fall" takes longer and the result is easier to see. There are also many videos online that show a feather and a heavier object falling together in a vacuum.

It is easy to prove mathematically that a large mass falls just as fast as a small mass (not counting air resistance). When an object falls, it falls faster and faster (it accelerates) because gravity continues to pull on it. According to Newton's Second Law, Force equals mass times acceleration, so acceleration = $\frac{\text{Force}}{\text{mass}}$. Let's drop two objects, one with mass m and one with mass 100m. The force of gravity on the first object is given by $F_1 = \frac{G \cdot m \cdot M_{Earth}}{\text{distance}^2}$, and the force of gravity on the second object is $F_2 = \frac{G \cdot 100m \cdot M_{Earth}}{\text{distance}^2}$. Assuming that both objects are at the same distance from the center of the earth, we can see that the force of gravity is 100 times larger for the heavier object. The acceleration experienced by the lighter object as it falls is $\frac{F_1}{m}$, while the acceleration of the heavier object will be $\frac{F_2}{100m}$. Since $F_2 = 100F_1$, the acceleration of both objects will be the same, and they will hit the ground at the same time.

The constant acceleration caused by gravity is called **g**, and it is approximately **9.81 m/s^2** near the surface of the earth. The acceleration due to gravity on the moon is 1.67 m/s^2.

Gravitational mass is measured by comparing the force of gravity on an object to the force of gravity of a known mass. This is usually done using a balance scale.

Inertial mass is measured by applying a known force to an unknown mass. The mass will accelerate according to Newton's Second Law, F = m a. We can measure the acceleration and use that to calculate the unknown mass.

There is no difference between gravitational and inertial mass.

FALLING OBJECTS

When Galileo used ramps to slow down the effects of gravity, he noticed that a ball that rolls down a ramp moves faster and faster. Measuring carefully, he found that if the ramp was four times as long at the same angle, the ball would only take twice as long to reach the bottom. If the ramp would be 9 times as long, the time to reach the bottom would only increase by a factor of 3. If you have the time for it, you could build your own ramp. Because timing rolling balls is difficult, you would need a very long shallow ramp, and average out several measurements for each height. Galileo didn't have a stopwatch, so one of the things he used for timing was his pulse.

Gravity works the same way without ramps, so if an object falls for 3 seconds as opposed to 1 second, we would expect it to fall 9 times as far.

Because of this squared relationship between time and distance, a graph that plots the position of a falling object over time has the shape of an upside-down parabola. (The basic equation of an inverted parabola is $y = -x^2$.) In fact, if you launch an object into the air at an angle, its rise and fall trace out a parabola. This is a good time to do a simple experiment. We can "launch" an object on a shallow ramp to slow the movement. You need a large flat surface, like a sturdy cardboard rectangle. Unlike for a regular ramp, the long ends will be the top and bottom. Roll a ball or marble upwards along the ramp at an angle from one of the bottom corners. You should see it following a parabolic path. Use a shallower ramp if you need to slow the motion more.

In the section "Measuring Motion", we saw that displacement is given by d = vt. We also know that acceleration is velocity/time. Acceleration measures how much faster the speed gets per unit of time, so it is usually measured in meters per second/second, or m/sec^2.

If $a = \frac{v}{t}$, then v = at.

Over the relatively short distances that are considered in your physics problems, the force of gravity doesn't change significantly. So, the constant force of gravity causes a falling object to experience a constant acceleration according to Newton's Law F = m a. Air resistance that would slow the object down is usually neglected.

As we saw earlier ("Important Formulas"), the displacement of an accelerating mass is given by:

$$\mathbf{d = v_0t + \frac{1}{2}at^2}$$

The acceleration due to gravity at the Earth's surface is about 9.8 m/s^2, or approximately 32 feet/sec^2. We use the letter g to indicate acceleration due to gravity, and g is the positive quantity 9.8 m/s^2. Since this acceleration is directed downward people commonly use –g. The displacement is

$$\mathbf{d = v_0t - \frac{1}{2}gt^2}$$

This displacement is the change in the vertical, or y-direction: $d = y - y_0$, where y_0 is the initial y position at time t = 0:

$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$

Already you can see that this is a quadratic equation. If you graph the position against the time t, you'll get a parabola as you would for any quadratic equation. This equation will work for any motion that involves a constant acceleration produced by some force
(Force = mass · acceleration).

If you drop a ball from height h, how long will it take to hit the ground? Well, h is the initial position, and the initial speed is zero. When the ball hits the ground, its position y will be 0:

$0 = h + 0t - \frac{1}{2} g t^2$

$\frac{1}{2} g t^2 = h$

$t^2 = \frac{2h}{g}$ so $t = \sqrt{\frac{2h}{g}}$

It should not be necessary to memorize this, but you can use it to check your calculations when you are looking at specific examples.

What is the impact velocity of the ball?

The velocity after time t is given by $v = v_0 + at$, and the initial velocity v_0 is 0, while the acceleration a is equal to -g. t is $\sqrt{\frac{2h}{g}}$ as we saw earlier.

$v = v_0 + at$

$v = 0 - gt$

$v = -g \sqrt{\frac{2h}{g}}$

$v = -g \frac{\sqrt{2h}}{\sqrt{g}}$ and since g divided by its square root is $\sqrt{g}$:

$v = -\sqrt{g}\,\sqrt{2h} = -\sqrt{2gh}$

If an object is launched vertically with a speed v_0, how long will it take before it hits the ground?

$h = v_0 t - \frac{1}{2} g t^2 + h_0$. We will assume that we start on the ground, so $h_0 = 0$.

$h = v_0 t - \frac{1}{2} g t^2$. When the object hits the ground h = 0

$0 = v_0t - \frac{1}{2}gt^2$

$0 = (v_0 - \frac{1}{2}gt)t$

This means that either t = 0 (the object was on the ground at time 0), or $v_0 - \frac{1}{2}gt = 0$. In the second case, $v_0 = \frac{1}{2}gt$, so $t = \frac{2v_0}{g}$.

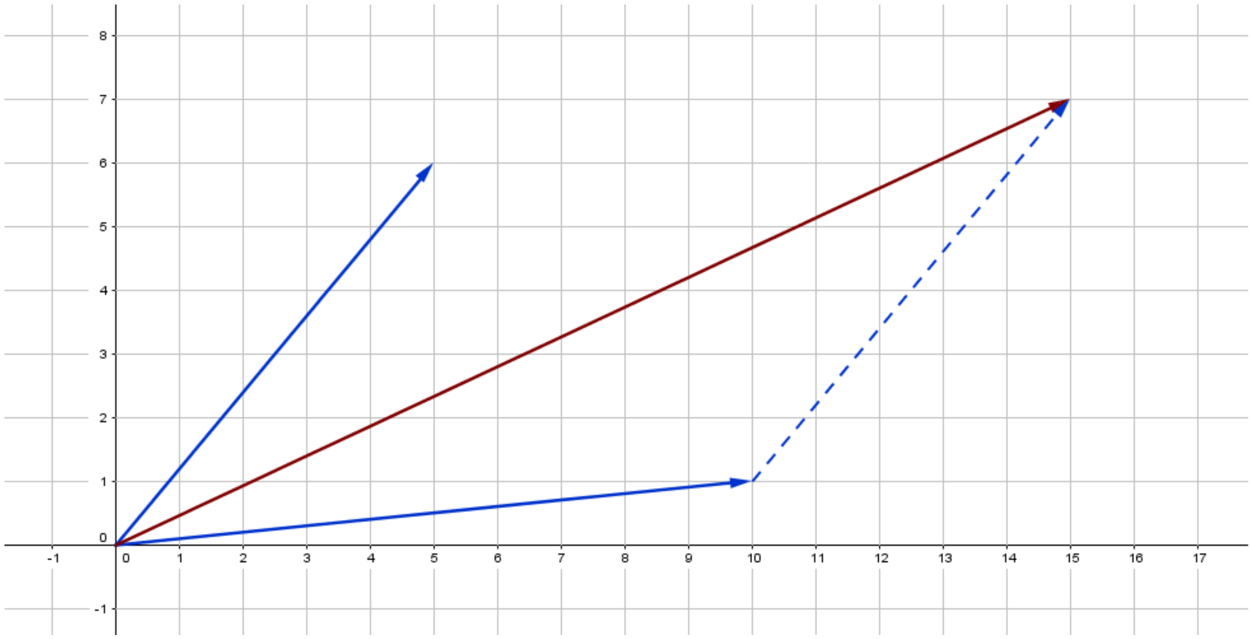
Interestingly, if you are standing at the edge of a cliff and throw a ball straight up with a velocity of, say 10 m/s, its impact velocity at the bottom of the cliff will be exactly the same as if you had thrown it straight down with a velocity of 10 m/s. This does make sense, because throwing the ball up at 10 m/sec will cause it to be falling back down with a speed of 10 m/s when it is level with the edge of the cliff. Although the impact velocity is the same, the time to impact is not.

VECTORS

Many problems only require you to consider a single dimension. Those problems may involve a falling object, or a particle moving along a straight line. In these cases you will usually be expected to use positive quantities for “up” or “to the right”, and negative quantities will be “down” or “to the left”. This follows the convention of using a single x-axis, or a single y-axis. Once we are dealing with more than one dimension, it becomes convenient to use vectors.

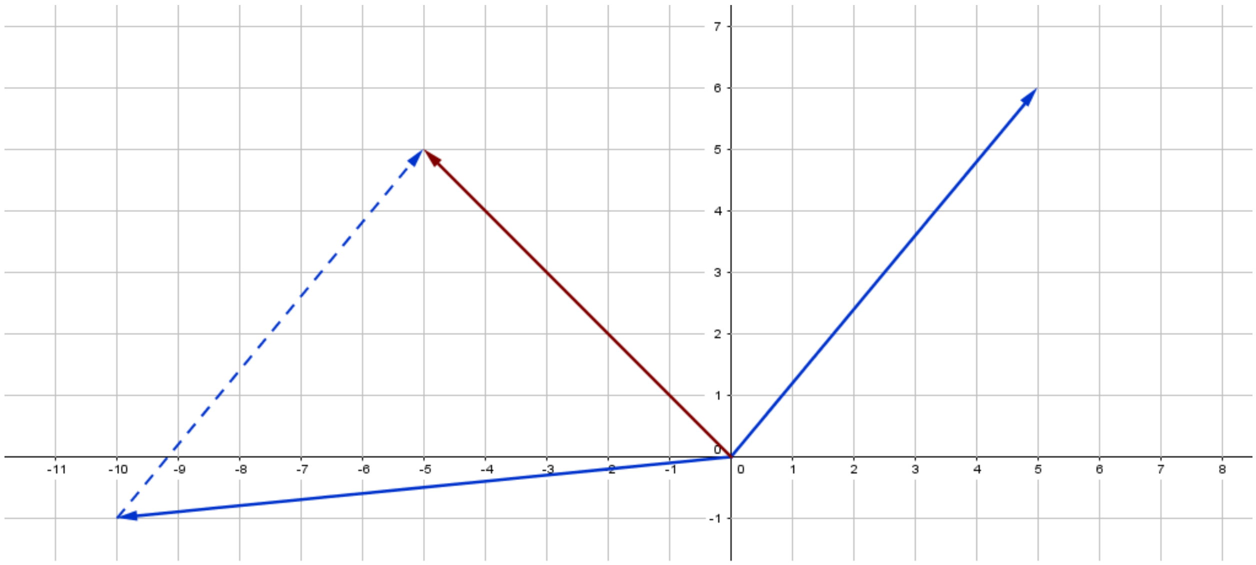
Vectors can be represented as arrows drawn from the origin to a point in a coordinate system. The point can have coordinates (x,y), for two dimensions, or (x,y,z) for three dimensions. A vector that runs from the origin to a point (x, y) is usually represented as <x, y>.

A vector can be broken down into separate x and y (or x, y and z) components. Each vector is the sum of its component vectors. A vector has both a magnitude and a direction. Any vector may be moved around and placed somewhere else on a plane or in a space, according to convenience. Two vectors may be added by summing up their x-coordinates and y-coordinates, or by placing them end to end to create a new vector.



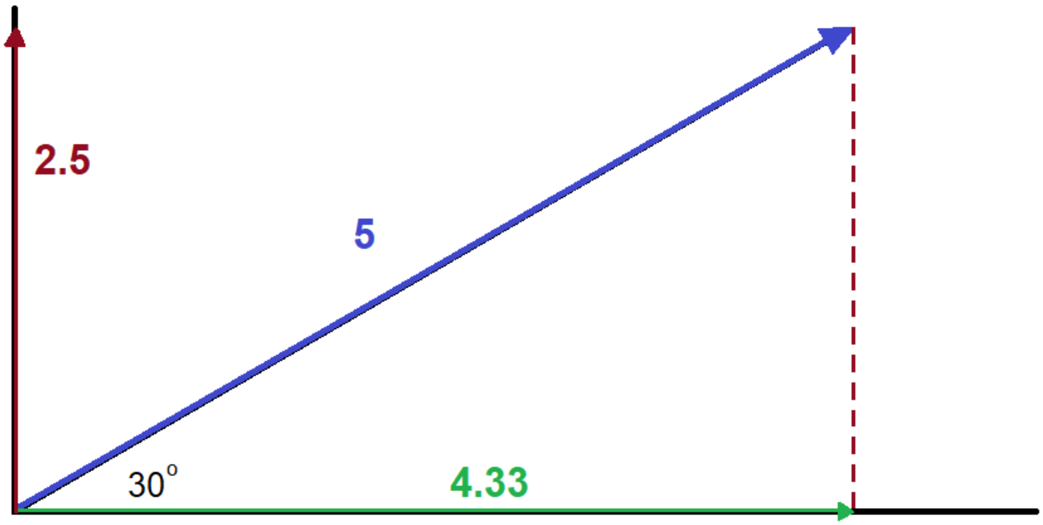
In the image above, the vector <10, 1> has been added to the vector <5, 6>. The resultant vector, shown in red, is <15, 7>. You can add coordinates, or imagine picking up one of the blue vectors and putting its tail against the tip of the other vector.

To subtract two vectors, take the vector that is to be subtracted and change it to point in the opposite direction (180 degrees). The length remains the same; only the direction changes. Then add the first vector to the opposite of the second vector. To subtract vector <10, 1> from vector <5, 6>, you need to flip it around and add it. <-10, -1> points in the exact opposite direction. When you add the vector <5, 6> to <-10, -1> you get the vector <-5, 5>:

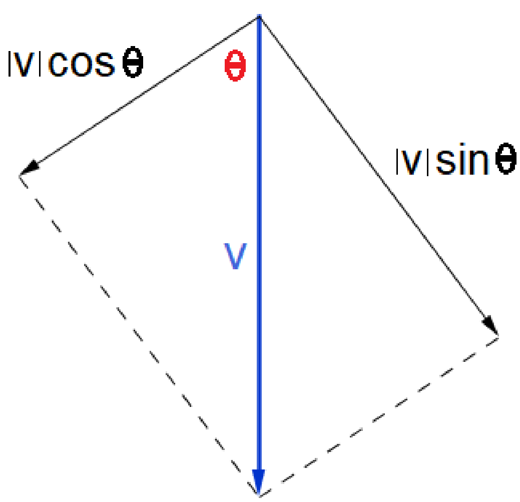


A force has both a “strength”, which is its magnitude (measured in Newtons), and a direction. This makes it convenient to use vectors to represent forces. The magnitude of the force is represented by the length of the vector. Another quantity that is best represented by a vector is velocity. Velocity can be either positive or negative and have a direction, usually specified by an angle. Speed on the other hand is a **scalar**, which is a numerical quantity without direction. A scalar may be positive or negative, but since there is no direction we can’t represent speed with a vector.

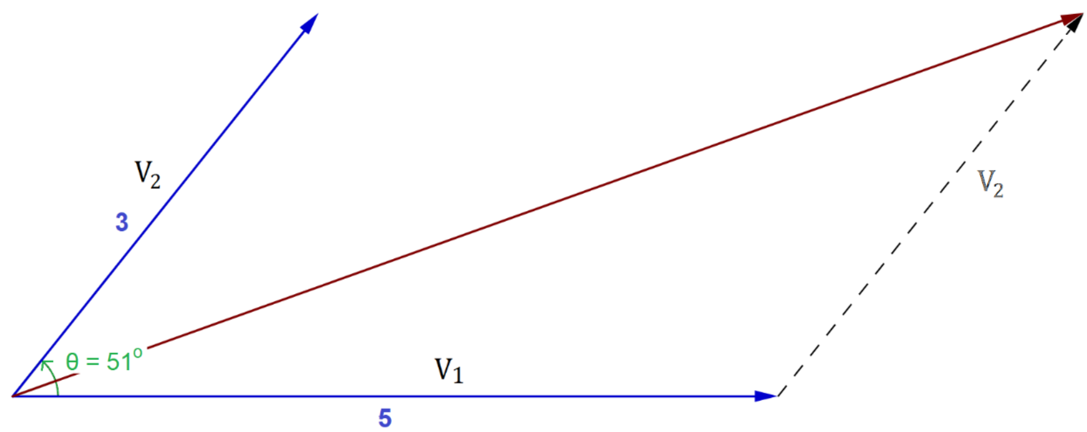
If two forces act on the same object the total force is the sum of the individual forces. If we represent the individual forces by vectors we can just add them to get a vector that represents the net force. To add two or more vectors just add their x and y components. You may have to find these components by using trigonometry. The x-component of a vector is the cosine of the angle between the vector and the x-axis multiplied by the size of the vector. The y-component is the size of the vector multiplied by the sine of the angle that it makes with the x-axis. For example, suppose a force of 5 Newtons acts at an angle of 30 degrees from a horizontal surface. This force can be represented by a vector with length 5. We can find the x-component of this force by taking the cosine of 30 degrees (about 0.866), and multiplying that by the length of the vector, 5, to get a horizontal component with a magnitude of 4.33. The y-component would be the sine of 30 degrees times 5, or 2.5. The blue vector can be represented as <4.33, 2.5>.



Each of the two components of the vector is a vector itself. The two component vectors add up to the original vector. In the same way, we can decompose a single force into two separate vectors to represent the magnitude of the force in two different directions. When you do that, just make sure that the x-coordinates of the two separate vectors add up to the x-coordinate of the original vector, and do the same with the y-coordinates. You will often be asked to decompose a force vector into two perpendicular components. This results in a rectangle that has the component vectors as two of the sides, and the original vector as the diagonal. In the image below, a downward pointing force has been randomly decomposed into two perpendicular components. Once you choose the direction of the first component, its magnitude and the direction and magnitude of the second component are already determined. You can then use the sine and cosine of a given angle to find the magnitude of the vectors. Just multiply the magnitude of the vector v, denoted by |v|, by the sine and cosine of the angle:



Here is a more complex problem, which you are not so likely to see. If you are given the magnitudes of two vectors, and the angle between them, you can find the resultant vector. Suppose that we have two vectors, v_1 and v_2, with a 51 degree angle between them. The magnitude of v_1 is 5 units, and the magnitude of v_2 is 3 units.



Although we can't change the direction of the vectors, we can imagine a slightly rotated coordinate system so that the lowest vector (v_1) lies exactly along the x-axis. The sum of the x-components is $|v_2|$ cos(51) + $|v_1|$, which is about (3)(0.62932) + 5 or 6.89. The sum of the y-components is $|v_2|$ sin(51) + 0, or approximately 2.33. Use the Pythagorean Theorem to get the length of the resultant vector, which is shown in red: $\sqrt{6.89^2 + 2.33^2}$ or approximately 7.27.

You can then use the inverse tangent of $\left(\frac{y}{x}\right)$ to obtain the angle of the resultant vector: $\tan^{-1}\left(\frac{2.33}{6.89}\right) \approx$ 18.7 degrees.

Alternatively, you can use the law of cosines. The angle between the dotted vector v_2 and the blue vector v_1 is supplementary to the given angle, so it must be 129 degrees. The red vector then represents c in the equation **$c^2 = a^2 + b^2 - 2ab \cos C$**, where C is the angle of 129 degrees. c = $\sqrt{5^2 + 3^2 - ((2)(15)(-.629))} \approx$ 7.27. The angle can be found from the equation $b^2 = a^2 + c^2 - 2ac \cos B$, which again works out to about 18.7 degrees.

If two vectors happen to be at right angles, you find the magnitude of the sum by using the Pythagorean Theorem, which is really the law of cosines when the angle is 90°. Once you have the magnitude, you still need the inverse tangent to find the direction of the resultant vector.

When you have a vector $\vec{V}_1$ and that vector changes a bit, we can represent the change by a small vector $\Delta\vec{V}$. A new vector $\vec{V}_2$ is created by adding the original vector and the vector representing the change: $\vec{V}_2 = \vec{V}_1 + \Delta\vec{V}$.

A vector may be multiplied by a scalar, as in $\vec{F} = m\vec{a}$. Force and acceleration have a direction, but mass is a scalar.

PROJECTILES

Force and velocity vectors that are at an angle from horizontal can be decomposed into horizontal and vertical components. If an object is launched at angle θ with velocity v, the vertical component is the magnitude of the velocity multiplied by the sine of the angle: $|v|\sin\theta$. The horizontal component of the velocity is $|v|\cos\theta$.

Each component of the velocity can be considered separately. The up and down movement of an object is affected only by its original vertical speed, and gravity. The vertical movement is completely independent of the horizontal movement. The vertical component is a parabola when graphed against time, and the steady horizontal velocity then creates a parabolic trajectory for the object. The upward component of the velocity determines how long the object is in the air, and the horizontal component determines how far it travels horizontally during this time.

First consider the vertical component of the motion. The vertical displacement is the height h, and the acceleration is downward with a magnitude of g. The initial velocity here is the vertical component of the velocity, $|v|\sin\theta$. We'll call that v_y

$d = v_0t + \frac{1}{2}at^2$

$h = v_yt - \frac{1}{2}gt^2$

The height h is 0 at the beginning and the end of the parabola. $0 = v_yt - \frac{1}{2}gt^2$.

$0 = (v_y - \frac{1}{2}gt)t$ so $t = 0$ or $v_y - \frac{1}{2}gt = 0$. Solve that last equation for t to get $t = \frac{2v_y}{g}$. The object is in the air for a total time of $\frac{2v_y}{g}$, where v_y is the vertical component of the launch velocity ($v_y = |v|\sin\theta$).

Because parabolas are symmetrical, maximum height occurs at half this time: $t = \frac{v_y}{g}$. To find the actual height at this point insert this value for t into the position equation:

$h = v_0\left(\frac{v_y}{g}\right) - \frac{1}{2}g\left(\frac{v_y}{g}\right)^2$

$h = \frac{v_y{}^2}{g} - \frac{1}{2} g \frac{v_y{}^2}{g^2}$

$h = \frac{v_y{}^2}{g} - \frac{1}{2} \frac{v_y{}^2}{g}$

$h = \frac{1}{2} \frac{v_y{}^2}{g} = \frac{v_y{}^2}{2g}$

Now look at the horizontal component:

To find how far the object travels in a horizontal direction, multiply the horizontal velocity by the total time the object is in the air ($\frac{2v_y}{g}$ from the previous section). d = vt. Since the horizontal velocity is $|v|\cos\theta$, the horizontal displacement is $|v|\cos\theta \cdot t$.

A projectile comes down in the same way that it goes up. The impact velocity is the same as the original velocity. If you need to find the original velocity, consider the vertical and horizontal components of the velocity as perpendicular vectors. You can use the Pythagorean Theorem to find the magnitude, and the inverse tangent to find the angle of both the original and the impact velocity.

WORK

W = F d

Work is defined as Force times distance. It is a scalar quantity (a number) rather than a vector. We multiply the force applied to an object by the distance the object moves. If there is a force exerted on an object but it does not move, the work done is zero by definition. Also, the force must continue to act on the object over a distance. Kicking a soccer ball does less work on the ball than you might think, because even though the ball ends up moving a long way you only count the distance moved during the time that your foot is in contact with the ball.

Even though work is a scalar quantity, it can still be negative. Negative work occurs when a force is applied in a direction that opposes the movement of the object. The work done causes the object to slow down. An example of this would be the work done by the force of friction that slows down a moving object. To spot invisible negative work, look at the speed (the magnitude of the velocity).

Since force is measured in Newtons, work is measured in **Newton-meters** in the SI system. Another name for a Newton-meter is a **Joule**.

1 Newton is 1 kg-m/sec^2, so 1 Joule is 1 kg-m^2/ sec^2.

The work done by gravity on a mass m that falls from a height h is **W = F h = m g h**.

In computing the work done, we only consider the component of the force that acts in the direction of the motion. For example, if you are pulling a heavy weight along the ground with a rope, you may be exerting a considerable amount of force on the rope. However, the force acts at an angle, and only its horizontal component actually moves the object. The amount of work done is **$W = |\vec{F}| \cos\theta \, d$**, where d is the displacement and θ is the angle between the force vector and the displacement vector. **If the force is perpendicular to the direction in which the object moves, no work is done** (cos θ is zero). If the angle θ is greater than 90 degrees the force is acting against the displacement, and the work done is negative.

NORMAL FORCE AND FRICTION

When an object rests on a surface, it exerts a force on that surface due to the action of gravity. In accordance with Newton's Third Law, the surface exerts an equal but opposite force on the object. This opposite force is called the **normal force**. The normal force is a contact force, and it is always perpendicular to the plane on which the object rests. Be very careful here when you do multiple choice questions, because the normal force and the force of gravity are NOT action-reaction pairs. The force of Earth's gravity on a mass is opposed by the force of the mass's gravity on the Earth. The force that a mass exerts on a surface is opposed by the force that the surface exerts on the mass. This difference becomes important when the surface is slanted.

For your problems in this section, you will determine the magnitude of the normal force by looking at the component of gravity that pulls the object into the surface. The normal force will be equal to mg on a level plane.

Friction opposes the movement of objects along a surface. The direction of the frictional force is parallel to the surface, opposite to the direction of movement. Friction is caused by electrical interactions at the molecular level. Friction is always proportional to the normal force (although it is perpendicular to it), and its magnitude also depends on the **coefficient of friction, μ**. This coefficient depends on the material involved.

Friction = $\mu \cdot F_{normal}$

Surprisingly, friction is not significantly dependent on the size of the contact surface. What matters is how hard the object pushes down onto the surface.

Static friction prevents an object from moving until a certain minimum force is applied. Static friction opposes the applied force. It is zero initially and increases linearly with the applied force until the object begins to move. To find the maximum possible amount of static friction, multiply the normal force by the coefficient of static friction, μ_s, as shown in the formula above. **Kinetic friction** acts while the object is moving. The coefficients of static friction are higher than those of kinetic friction.

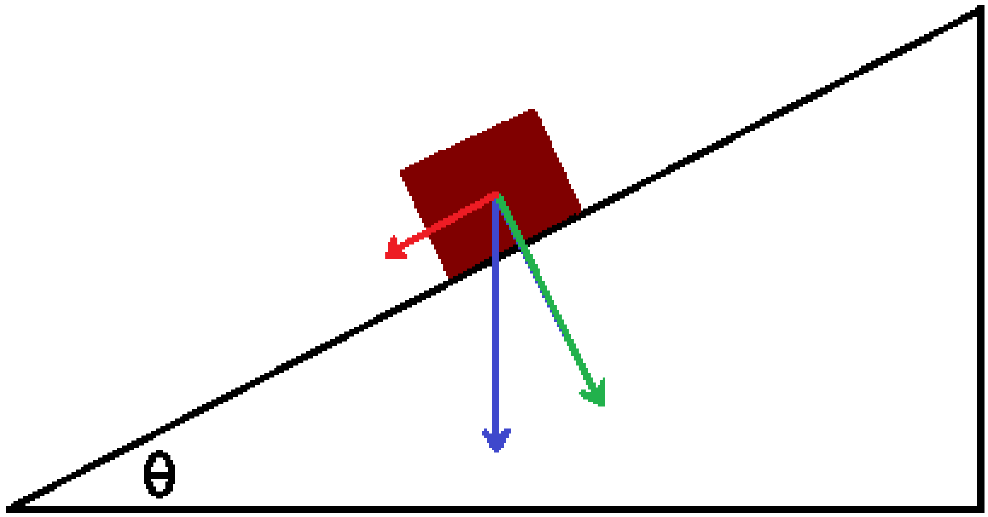
$F_{Static\ friction} \leq \mu_S F_N$

$F_{Kinetic\ friction} = \mu_K F_N$

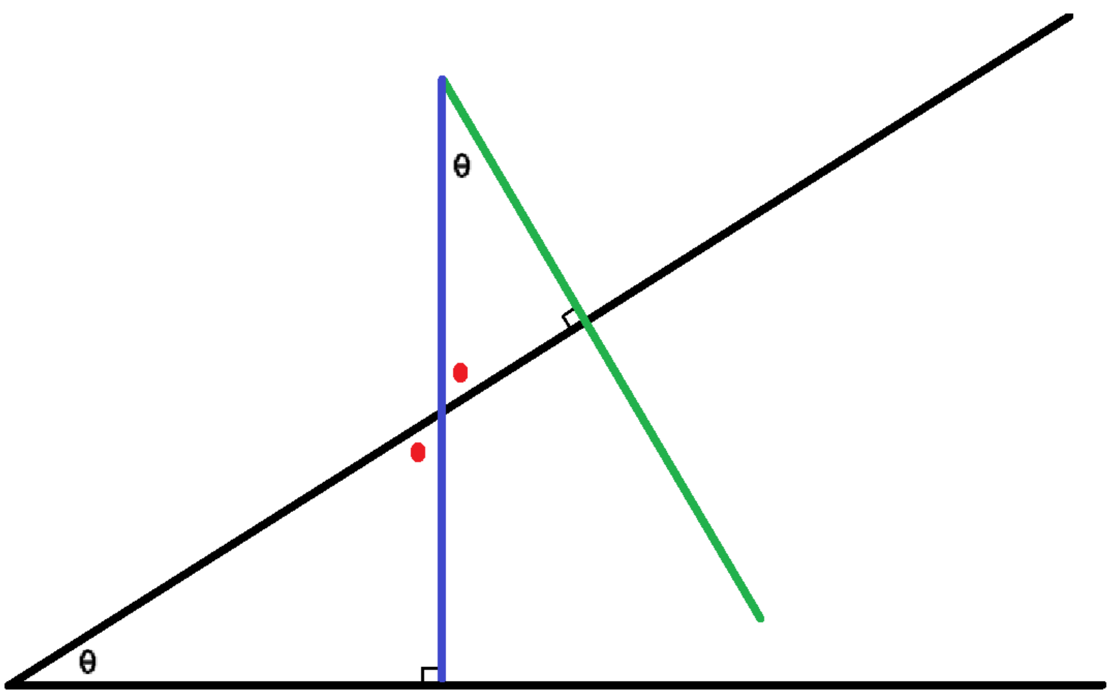
Newton's First Law says that a moving object remains in motion unless a force acts on it. **Whenever a problem states that an object is moving at a constant speed, that means there is no net force acting on it!** If you slide a box along the floor *at a constant speed*, the force you are applying is exactly equal and opposite to the force of friction between the box and the floor. Use this idea to calculate the force of friction.

RAMPS

If a mass is located on an inclined plane, the force of gravity on that mass is still mg as expected. However, we should decompose the gravitational force into two components to help us understand the forces experienced by the mass. One of these components should point down the plane, the way the object would slide down. If the plane is at an angle θ to the ground, it works like this:



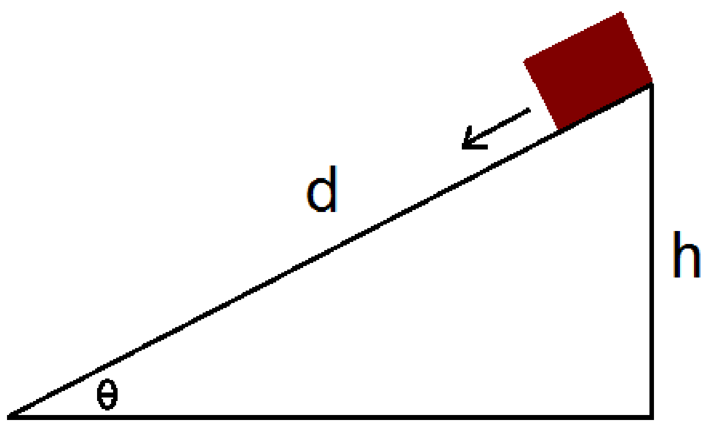
The blue vector represents the force of gravity, and the green and red vectors are its components. The force of gravity on the mass is equal to mg. One component of the force of gravity is perpendicular to the plane (the green vector).



As shown above, the two angles marked with red dots are equal because they are vertical angles (see geometry). The angle of the ramp, which is bottom angle marked θ, and the angle marked with the red dot add up to 90 degrees because the triangles in the picture are right triangles. Because the angles marked with red dots are equal, the angle between the blue vector and the green vector is the same as θ, the angle of incline of the plane.

The magnitude of the blue vector is equal to the force of gravity, mg, while the magnitude of the green vector is mg cos θ. The green vector is a measure of how hard gravity pulls the mass directly into the plane. This causes the mass to exert a force on the plane. By Newton's Third Law, this causes the plane to exert an equal and opposite force on the mass, which is the normal force.

The other component of the gravitational force pulls the block directly down the incline (the red vector). Its magnitude is mg sin θ. It is opposed by the force of friction between the plane and the block. Most planes in your course will be frictionless, but if not you'll have to find the force of friction by looking at the magnitude of the normal force.



If the mass starts at the top of the incline at height h and gravity pulls it to the bottom, the work done is Force, mg sin θ, times the distance. Since $\sin\theta = \frac{h}{d}$, the distance d is equal to $\frac{h}{\sin\theta}$. So, work = $mg\sin\theta \cdot \frac{h}{\sin\theta} = mgh$. This is the same amount of work that gravity would do on the mass if it fell directly from height h. This is important to note, because if you need to calculate the final speed of an object that is sliding down a frictionless incline, you can just treat it as if it simply fell from the top of the ramp. Gravity has the same effect either way. Gravity is a **conservative force**, because the amount of work done is the same regardless of the path taken. (An example of a non-conservative force is friction, as the amount of work done by friction is larger when the path is longer.)

ENERGY AND WORK

Energy is required to do work. Energy can be changed from one form into another, but it is always conserved. **Kinetic Energy** is the measure of the energy contained in a moving object. the actual amount of kinetic energy depends on both the mass and speed of the object. **Potential energy** is "stored energy". Much of your course looks at **mechanical energy**, which is the sum of kinetic energy and potential energy. Mechanical potential energy can be in the form of gravitational potential energy and/or elastic potential energy. Gravitational potential energy is based on the location of an object within a gravitational field. When you lift an object up you give it more gravitational potential energy. Elastic potential energy is the energy that is stored in an object like a stretched rubber band or a compressed spring.

The Law of Conservation of Energy says that in a closed, isolated system, energy is conserved.

From an energy point of view, there are two types of forces:

Conservative forces do the same amount of work regardless of the path taken by an object. Conservative forces such as gravity, magnetic force, electrical force, and the force exerted by a spring can change potential energy to kinetic energy, or vice versa, but the total amount of mechanical energy stays the same.

Non-conservative forces such as applied force, friction, air resistance, the normal force, and tension are forces that can change the mechanical energy of an object. Energy is always conserved, but mechanical energy is often converted into heat energy. The work done by such non-conservative forces depends on the path taken.

Gravity is a conservative force that can change potential energy into kinetic energy. When an object falls, it loses potential energy but gains kinetic energy. The potential energy of the object is mgh Joules (equal to the work done by gravity to make the object fall), which is converted into kinetic energy ($\frac{1}{2}mv^2$ Joules) as it falls. As an example, consider gravity acting on an object located 1 meter above the ground. The potential energy depends on the height, and the work done by gravity to bring the object to ground level, mgh, is the same whether it just falls or slides down a ramp.

Note that in this case we are setting the zero point of gravitational potential energy at ground level. That isn't really the absolute zero point, since something could fall down a well. It doesn't really matter because here we are only concerned with changes in potential energy at relatively small distances. You can take the zero point for mgh at a tabletop, your outstretched hand when you catch a ball, or anywhere that is convenient for your problem.

If you just drop an object, the initial speed is 0 and mgh = ½ mv^2. Because m appears on both sides of the equation, gh = ½ v^2, which makes it really easy to calculate the impact speed of the object.

If you throw a 2kg ball upwards with a speed of 10m/sec, the initial kinetic energy of the ball is ½ mv^2 = 100 kg $meter^2/sec^2$. The unit for this is Newton-meters, or Joules. Recall that 1 Newton is the force that gives a 1 kg object an acceleration of 1 $meter/sec^2$. When that force has moved the object a distance of 1 meter, one Joule of energy has been used. To get a 2 kg ball moving at a speed of 10 m/sec requires 100 Joules of energy. The ball will rise to a maximum height, at which point all of its kinetic energy has been converted to gravitational potential energy. It now has 100 Joules of potential energy, mgh. 100 = $|m\ g\ h|$= 2 x 9.8 x h. From this we can calculate that the maximum height must be 5.102 meters (above the starting point). Since $v = v_0 + at$, we can also calculate that achieving maximum height and a velocity of 0 will take about 1.02 seconds. When the ball has come back down to its original height, its speed is 0 – gt = - 10m/sec. Its kinetic energy is again 100 Joules. So, if you throw a ball upwards with a certain speed, it will have that same speed when it comes down to its starting point. If it then continues to fall further, past the point where it was thrown up, it will gain additional kinetic energy as the force of gravity converts potential energy into more kinetic energy. Calculate the net change in the ball's kinetic energy like this: $m\ g\ h = ½\ m\ v^2 – ½\ m\ v_0^2$.

A more powerful engine can accelerate a car faster. **Power** is work done per unit of time. The power of a car engine is measured in horsepower, but physics normally uses **Watts**. One Watt is 1 Joule/second (work per unit of time), and 1 horsepower is about 745.7 watts. The electrical power used by lightbulbs is measured in watts. A 60 Watt incandescent lightbulb uses 60 Joules every second, or 216,000 Joules every hour. Its glowing metal filament wastes a lot of energy as heat. You might want to replace that with a much more efficient and LED (light emitting diode) bulb that uses only 8 or 9 Watts.

Just how much energy is contained in a mass m moving at velocity v? The best way to see that is to think how much energy (work) is required to stop the mass. Often this work is done by friction, and the object appears to stop by itself. Work is equal to Force times distance, and F = ma. The total work done by this force is ma · d. In this case the work will be negative, since the force acts in the opposite direction from which the object is moving. That will be accounted for by the negative acceleration, but to simplify things we'll only look at magnitudes here. At first the velocity is v, and when the mass is stopped the velocity is 0. The mass will decelerate at a constant rate, so to get the distance traveled by the mass while the force acts, we can use the average velocity. That average is the initial velocity v, plus the final velocity, 0, divided by 2: $\frac{v+0}{2}$. The mass will travel a distance of (average velocity) x time = $\frac{v}{2}$ t. The work needed to stop the mass can now be written as ma · $\frac{v}{2}$ t. The acceleration a is the change in velocity over time, and in this case the velocity changes from v to 0. That means v actually is the magnitude of the change in the velocity here: a = $\frac{v}{t}$. Substituting that into ma · $\frac{v}{2}$ t, we see that the work done is m · $\frac{v}{t} \cdot \frac{v}{2}$ t , or ½ mv^2. Before the object stopped, it had energy of motion (kinetic energy), but after we do the work of stopping it, that energy is zero.

Kinetic Energy = ½ mv^2

In the same way, we can calculate the work done to speed up a mass moving at an initial velocity v_0 by applying a constant force over a distance d, until the mass reaches a final velocity v:

Work = F · d

Work = ma · d

a = $\frac{v-v_0}{t}$ and d = v_{ave}· t

The average velocity is $\frac{v+v_0}{2}$, so we can say that d = $\frac{v+v_0}{2} \cdot t$

Putting that together we get

Work = $m \cdot \frac{v-v_0}{t} \cdot \frac{v+v_0}{2} \cdot t$

t cancels out, and $v - v_0$ times $v + v_0$ is the difference of two squares:

Work = $m \cdot \frac{v^2 - {v_0}^2}{2}$

Rearrange that to make it correspond to the expression for kinetic energy:

Work = $\frac{mv^2 - m{v_0}^2}{2}$

Work = $\frac{1}{2}$ m v^2 – $\frac{1}{2}$ m v_0^2

This tells you that work done on an object is equal to the change in its kinetic energy. That fact is referred to as the **Work-Energy Theorem**. Work is measured in Joules, and so is energy.

Since this equation relates work to velocity, it is possible to calculate the velocity of a mass if you know the work done on it. Note that the work could be done by gravity, which provides an easy way to calculate the velocity of a falling object. In fact, many of the gravity kinematics problems can be solved by using the equation

Work = F d = m g h = $\frac{1}{2}$ m v^2 – $\frac{1}{2}$ m v_0^2

Because m appears on both sides of the equation m g h = $\frac{1}{2}$ m v^2 – $\frac{1}{2}$ m v_0^2 it can be eliminated. The speed of a falling object does not depend on its mass – lighter objects fall just as fast as heavier ones:

gh = $\frac{1}{2}$ v^2 – $\frac{1}{2}$ v_0^2

This equation is equivalent to something we saw earlier when we rearranged the motion equations: **ad =** $\mathbf{\frac{v^2 - v_0^2}{2}}$. The acceleration a is g as the object falls, and the height h is the displacement d.

<u>Calculus:</u> The work equation can also be derived using calculus, just in case the work is not done by a constant force. Divide the distance d into infinitely small portions dx. For each infinitely small distance, the work done is Fdx, which is ma dx. The acceleration over this distance is $\frac{dv}{dt}$, to give dW = m $\frac{dv}{dt}$ dx

Rearrange this to dW = m $\frac{dx}{dt}$ dv, because $\frac{dx}{dt}$ represents the regular-sized variable v, the velocity over the infinitely small distance dx. Now we can integrate the expression dW = mv dv to get the formula for the work done in terms of the velocity:

$$W = \int_{v_0}^{v_f} mv\, dv = m\int_{v_0}^{v_f} v\, dv = \tfrac{1}{2} m v_f^{\,2} - \tfrac{1}{2} m v_0^2$$

Roller Coasters

Roller coasters provide an example of several important physics principles. A steep drop followed by a climb uphill shows how gravitational potential energy is converted into kinetic energy, and then back into gravitational potential energy. Some energy is always lost due to friction, so unless extra power is provided the starting point of the ride is the highest point, and you lose height as you go.

Typical problems of this type show a starting height and a lower second height, and ask you to calculate the speed. Conversely, you may be asked to calculate the height from the given speed. Friction is ignored. The important thing here is to keep in mind that mechanical energy (the sum of potential and kinetic energy) is always conserved.

If the second height is lower than the first the gravitational potential energy has decreased by an amount directly determined by the difference in height. While you could calculate the total gravitational potential energy at the first height, and subtract the total at the second height, usually you only need the difference. For example, if the second position is 3 meters lower than the first, the decrease in potential energy is mg times 3, which is then equal to the gain in kinetic energy: $3mg = \frac{1}{2} mv^2$. Be careful though, as the gain in kinetic energy doesn't directly correlate to the speed. If the car has an initial speed, you must calculate the initial kinetic energy and add the amount gained. Then use the total kinetic energy to determine the final speed.

Escape Velocity

What goes up must come down – or will it? We already know that it is possible to launch objects that leave Earth, so what velocity would that need?

We can calculate the required velocity based on the conservation of energy. As an object is launched upwards it gains gravitational potential energy, but loses kinetic energy. Because the pull of gravity gets weaker and weaker as the object moves away from the center of the Earth, it is possible for it to overcome the gravitational pull. Once it is a (theoretical) infinite distance away from Earth, the gravitational potential energy will be zero. If we gave the object just enough velocity to escape, then all of the kinetic energy will be used up by the time it reaches this point. The kinetic energy will be zero, just like the gravitational potential energy. Since the final mechanical energy (the sum of the gravitational potential energy and the kinetic energy) is zero in the end, it must also be zero to start with.

To do this calculation, we must consider the actual gravitational potential energy at the surface of Earth. Normally we do relative measurements and set the zero point at the Earth's surface. Here we will actually consider the force of gravity on an object with mass m at the surface of the Earth, which is $\frac{\mathrm{G \cdot mM_e}}{\mathrm{R^2}}$, where R is the radius of the Earth and M_e is the mass of the Earth. The kinetic energy at launch is ½ mv^2, while the gravitational potential energy is mgh. In this case the height h is the radius of Earth. Just imagine our planet retaining its mass while shrinking to the size of a pea, and the object floating in space at a distance R from that pea. Since force = mass x acceleration, we can replace mg by the force of gravity, $\frac{\mathrm{G \cdot mM_e}}{\mathrm{R^2}}$, and the height h is equal to R. The magnitude of the gravitational potential energy is $\frac{\mathrm{G \cdot mM_e}}{\mathrm{R^2}} \cdot$ R or $\frac{\mathrm{G \cdot mM_e}}{\mathrm{R}}$. That potential energy has a minus sign, because it gets larger as R increases: $-\frac{\mathrm{G \cdot mM_e}}{\mathrm{R}}$

The initial mechanical energy is equal to the final energy, which was zero:

$\frac{1}{2} mv^2 + -\frac{\mathrm{G \cdot mM_e}}{\mathrm{R}}$ = 0 (final kinetic energy) + 0 (final gravitational potential energy)

$\frac{1}{2} mv^2 = \frac{\mathrm{G \cdot mM_e}}{\mathrm{R}}$

$\frac{1}{2} v^2 = \frac{\mathrm{G\ M_e}}{\mathrm{R}}$

$v = \sqrt{\frac{\mathrm{2G\ M_e}}{\mathrm{R}}}$

That works out to about 11,186 m/s, which apparently is not a practical launch velocity due to drag and heating effects caused by Earth's atmosphere. We can however launch something with enough velocity to get it into orbit, and then accelerate it in space where the force of gravity is weaker.

<u>*Calculus:*</u>

We can use calculus to determine the work that gravity does to change the kinetic energy of an object launched from Earth's surface to zero. Although the force of gravity changes constantly along the trajectory, we can divide the distance r up into infinitely many parts dr. That work will be negative because it is opposite the direction of motion. Work = Force of gravity x distance, or $-\int \frac{\mathrm{G \cdot mM_e}}{\mathrm{r^2}}\,\mathrm{dr}$.

We want to integrate that starting at the radius of the Earth, R, all the way to infinity. To get to that point we set the upper bound of the integral to some random variable, like maybe h, and then let h go to infinity:

$-\int_R^h \frac{\mathrm{G \cdot mM_e}}{\mathrm{r^2}}\,\mathrm{dr}$, *which is* $-\int_R^h (\mathrm{G \cdot mM_e}) r^{-2}\, dr$.

That solves as $GmM_e r^{-1}\Big[{}^{h}_{R} = \frac{\mathrm{G\ mM_e}}{\mathrm{h}} - \frac{\mathrm{G\ m\ M_e}}{\mathrm{R}}$. As h goes to infinity the limit is $-\frac{\mathrm{G\ mM_e}}{\mathrm{R}}$. The work done is equal to the change in kinetic energy, which is 0 – $1/2mv^2$, and again v works out to $\sqrt{\frac{\mathrm{2G\ M_e}}{\mathrm{R}}}$.

Springs

Springs can store elastic potential energy. The force required to compress or extend a spring depends directly on the distance that the spring is compressed or extended, provided that you don't extend it too far:

F = kx

where x is the distance past the resting length of the spring, and k is the spring constant that depends on the properties of the spring itself. Once you extend or compress a spring, it has elastic potential energy that can be converted into kinetic energy. To calculate the amount of elastic potential energy, we can imagine a mass m attached to a spring that has been compressed a distance x. When we release the spring, it will exert a force on the mass. However, when it is first released that force is large, but by the time the spring reaches its resting length the force is zero. Fortunately the force decreases at a steady rate, so we can use the average: $F_{ave} = \frac{F_0 - F}{2} = \frac{F_0 - 0}{2}$. Since the initial force is given by F = kx, the average force will be ½ kx. This force does work on the mass m, by acting over a total distance x. Work = F · d = ½ kx · x. The elastic potential energy that is available to do work is ½ kx^2.

For more details on springs check the section on Simple Harmonic Motion.

MOMENTUM

As soon as the first people started throwing rocks at each other, they realized that a large object is more dangerous than a small object moving at the same speed. Once they became more civilized, they invented ways to make small projectiles equally lethal by shooting them at high speed using bows and later guns. The physics details behind this are actually somewhat complicated.

Momentum is the "amount of motion" of an object. It is determined by both mass and velocity.

Momentum = mass x velocity = mv

From this, you can see that momentum is not the same as kinetic energy, which is ½ mv^2. If two objects have the same momentum but different weights, the lighter object has the greater velocity, and therefore more kinetic energy. As our civilization has progressed, experts now use physics principles to argue over whether we should favor momentum or kinetic energy in our ongoing quest to produce more lethal projectiles.

Newton's second law was originally phrased in terms of momentum. Let's see how that works.

Force = m · a

The acceleration a is the change in velocity over time. We can indicate the change in anything by using the symbol Δ. So, a = $\frac{\Delta v}{t}$, and

Force = m · $\frac{\Delta v}{t}$

Force = $\frac{m \Delta v}{t}$

Because the mass m doesn't really change at ordinary speeds, we can say that force is equal to the change in momentum over time:

Force = $\frac{\Delta \text{ momentum}}{\text{time}}$

Impulse

A force may act on an object for a brief time, or for a long period. The **impulse** is the product of the force and the time, Ft, and it is equal to the change in momentum. In simple terms, where the initial velocity is zero:

Ft = m a

Ft = m · $\frac{v}{t}$ · t

Ft = m v   (The impulse is equal to the momentum)

If the initial velocity is not zero, we can say that the impulse is the change in momentum:

Ft = Δ momentum

Most collisions involve a sudden change in momentum over a short time. To reduce the force involved, cars are designed to crumple on impact so that the time is greater and the force is less.

If the force is not constant, the total impulse may be calculated by finding the area under a force – time graph. If that graph is not a straight line we can use calculus to get the impulse.

Collisions and Conservation of Momentum

There are two types of collisions between objects: **elastic collisions** and **inelastic collisions**. Regardless of the type of collision momentum is always conserved. Sometimes two objects collide and both stop. Momentum, like velocity, is a vector quantity, and the momentum of one object can cancel out the momentum of the other if they have opposite directions. By convention we normally assign a positive velocity to an object moving to the right, and a negative velocity to an object moving to the left.

The total momentum before the collision is equal to the total momentum after the collision, within an isolated system (no net external force such as friction), even if energy is lost due to factors like sound and heat produced. So how does the universe know to adjust the velocities of two separate objects so that momentum is conserved? Well, here is where Newton's third law comes in. During a collision between object A and object B, A exerts a force on B. Inevitably, B will exert an equal and opposite force on A. These forces act for the same period of time t, which is the brief time of the collision. Since we just saw that the impulse Ft = Δmv, the change in momentum is equal for both objects. As a result, there is no net change in momentum due to the collision.

When you do problems that ask you to calculate the change in momentum of an object, like a ball bouncing off a wall, be careful with those minus signs! Suppose a 1 kg ball moves directly to the right at 4 m/sec, hits a wall, and then bounces off directly to the left at 3 m/sec, the change in momentum is NOT equal to 1 kg m/sec. Always calculate a change by taking the new value and subtracting the old value:

$\Delta mv = mv_{final} - mv_{initial}$

Δmv = -3 kg m/sec – 4 kg m/sec = -7 kg m/sec.

The magnitude of the change is 7 kg m/sec, which is 7 N/sec. Because impulse = change in momentum, the impulse equals 7 N/sec.

Sometimes collisions involve an angle, such as a ball hitting a wall at an angle and bouncing off at an angle. In those cases, consider that momentum is conserved in both the vertical and horizontal directions.

Kinetic energy is also conserved in perfectly elastic collisions. However, perfectly elastic collisions are rare. Usually the total kinetic energy will be less after a collision than before, because some of the energy is converted to other forms, like sound. In a perfectly *inelastic* collision the two colliding objects stick together and the maximum possible amount of kinetic energy is lost.

It may seem strange that a lot of kinetic energy would be lost in a collision, and yet the total momentum stays the same. To really understand this we need to consider the **center of mass**.

Center of Mass

In physics, we find that objects behave as if all of their mass is concentrated at some center point. This is the **center of mass**. We use the idea of center of mass when we determine gravitational force. In the equation $F_{Gravity} = \frac{G\,M_1M_2}{\text{distance}^2}$ we use the distance between the center of the earth and the center of another object, **not** the distance between the earth's surface and the object.

Because an object may have an odd shape, and its density may not be uniform, it is possible for the center of mass to be located outside the object.

So how can we calculate the position of the center of mass, given the position of its parts? Well, let's use the example of a weighted average grade. Suppose your homework counts for 25% of your grade, quizzes for 35%, and your final exam counts as 40% of your grade. If you have a grade of 90 on your homework, you would multiply that grade by .25. Then you would take your quiz grade and multiply that by .35, and your final exam grade by .4. That would give each grade the proper weight.

You can do the same with the x-coordinates of each part of a mass. Suppose we have two masses, one 12 kg mass at position 2, and one 4 kg mass at position 7. The center of mass should be somewhere between position 2 and position 7, but closer to 2 since the first mass has more weight. Unfortunately we are not given that weight as a convenient percentage, so we have to calculate that ourselves. The 12 kg mass accounts for $\frac{12}{12+4}$ or .75 of the total weight. The 4 kg mass accounts for $\frac{4}{12+4}$ or .25 of the total. To get the average x-coordinate, multiply each x-coordinate by its relative weight:

$x_{ave} = (.75)\, x_1 + (.25)\, x_2$

$x_{ave} = (.75)\, 2 + (.25)\, 7$

$x_{ave} = 3.25$

In general, we get x_{ave} like this:

$$x_{ave} = \left(\frac{m_1}{m_1 + m_2}\right) x_1 + \left(\frac{m_2}{m_1 + m_2}\right) x_2$$

$$x_{ave} = \frac{m_1 x_1}{m_1 + m_2} + \frac{m_2 x_2}{m_1 + m_2}$$

$$\mathbf{x_{center} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}}$$

where x is the position (the distance from some arbitrary origin). Notice that this is the same as saying that the total mass times the position of the center equals $m_1 x_1 + m_2 x_2$.

All of this works exactly the same for the velocity of the center of mass. Multiply the velocity of each part by the percentage of the mass that each part represents:

$$v_{ave} = \left(\frac{m_1}{m_1 + m_2}\right) v_1 + \left(\frac{m_2}{m_1 + m_2}\right) v_2$$

$$v_{ave} = \frac{m_1 v_1}{m_1 + m_2} + \frac{m_2 v_2}{m_1 + m_2}$$

$$\mathbf{v_{center} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}}$$

This equation also tells you that the total momentum of the system is the sum of each separate momentum because it rearranges like this:

$(m_1 + m_2)v_{center} = m_1 v_1 + m_2 v_2$

$mv = m_1 v_1 + m_2 v_2$

When two objects collide, momentum is always conserved, even if energy is lost. That may not seem as odd if you look at the two objects as a single system, with a center of mass. Even though the objects collide, there is no external force on the system. From the point of view of the center of mass, nothing significant has happened and it just continues along with the same velocity as before. In order to change the velocity of the center of mass, a force is required, and that is not there. The interaction of the collision is inside the system only. When you consider a system of objects, it makes sense to look at forces as internal or external. **Internal forces** are forces that act between objects in the system. They cannot do work on the system and cannot change the total energy of the system. **External forces** are outside forces that act on the system. They can do work on the system and transfer energy into or out of the system.

If an external force does act, we can find the acceleration of the center of mass:

$F = ma = m_1a_1 + m_2a_2$

$F = (m_1 + m_2)a = m_1a_1 + m_2a_2$

$a = \frac{m_1a_1 + m_2a_2}{m_1 + m_2}$

The lower an object's center of mass, the greater its stability. More force is required to tip it over.

If you are taking a basic physics course, this will probably conclude all of the topics you will cover.

CIRCULAR MOTION

Circular motion can apply to an object moving in a circular path (revolution), or to a particular point on a rotating object. During **uniform circular motion** the speed stays constant. However we will see that even though the speed is constant, the velocity (speed with direction) changes so that there is still acceleration.

How fast is something going when it is moving in a circle? Consider a solid wheel spinning on an axis. A point on the wheel close to the edge moves much faster than a point close to the center.

For each particular point we select at a radius r from the center, the velocity can be found by taking the distance moved, 2πr, and dividing that by the time it takes for the point to go completely around the circle. This is the linear velocity, which is called the **tangential velocity** in the case of circular motion. If the force that makes an object constantly turn in a circle is suddenly removed, it will continue along a straight line tangent to the circle with the same speed, according to Newton's first law. This calculation of linear velocity is not always helpful to describe the speed. A more natural way to state the speed of an object in circular motion is to use revolutions per minute (**rpm**).

But what about a door, which never makes a complete revolution? It is actually best to consider the angle of rotation rather than whole revolutions. θ is the angle of rotation, also called the **angular displacement**. θ is best measured in radians. 1 radian is the angle created when 1 radius laid along the edge of the circle. The circumference of a circle is 2π times the radius, so there are 2π radians in a circle. 2π radians = 360 degrees, which means that 1 radian is just over 57 degrees. If you know θ in radians then for any given point on the rotating object that is a distance r from the center you can find the distance that the point travels. This distance is the **arclength**, S.

Let's find S for an object that moves in a circular path with a radius of 4 meters, when the angular displacement is 1.2 radians. The entire circumference of the circle is 2πr, but S is only a fraction of that distance. In fact it is $\frac{1.2}{2\pi}$ radians. You have to multiply the total circumference, 2πr, by this fraction, which cancels out 2π and leaves 1.2 radians times r. S = 1.2 x 4 = 4.8 meters.

In general, the arclength is found by multiplying the angle in radians by the actual length of the radius. **$S = r\theta$**. Notice that S and r are both lengths, while radians are just numbers. The radian is considered a "unitless" unit.

Angular velocity (ω) is a measure of how fast the angular displacement is changing.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

(Δ stands for "the change in"). Angular velocity is measured in radians per second. It can be found directly if you can figure out how fast θ is changing. To convert angular velocity to linear velocity, you need to consider the actual distance that the object moves when the angle changes by one radian. That distance depends on the radius. For example, if an object is moving in a circle at a rate of 2 radians per minute, and the circle has a radius of 5 meters, then the linear speed is 10 meters per minute, because each radian represents 5 meters in this situation. To find the linear speed, simply multiply the angular speed by the length of the radius:

$v = \omega r$

That also that means that $\omega = \frac{v}{r}$.

Consider a spinning disc. A point close to the outside edge of the disc moves faster than a point near the center, because linear velocity depends on the radius: $v = \omega r$. However, angular velocity does not depend on the radius. Every point on a rotating object has the same angular velocity.

If the angular velocity changes over time, we can find the average angular velocity just like we do for regular velocity: average angular velocity = $\frac{\omega_{final} - \omega_{initial}}{2}$.

Angular acceleration (α) measures how fast the angular velocity is changing. **$\alpha = \frac{\Delta\omega}{t}$**

If there is angular acceleration, the tangential (linear) velocity is of course also changing. That means that there is also linear acceleration, a. We already saw that $v = \omega r$, so since $a = \frac{\Delta v}{t}$, and $\alpha = \frac{\Delta\omega}{t}$, you can multiply α by r to get the linear acceleration a:

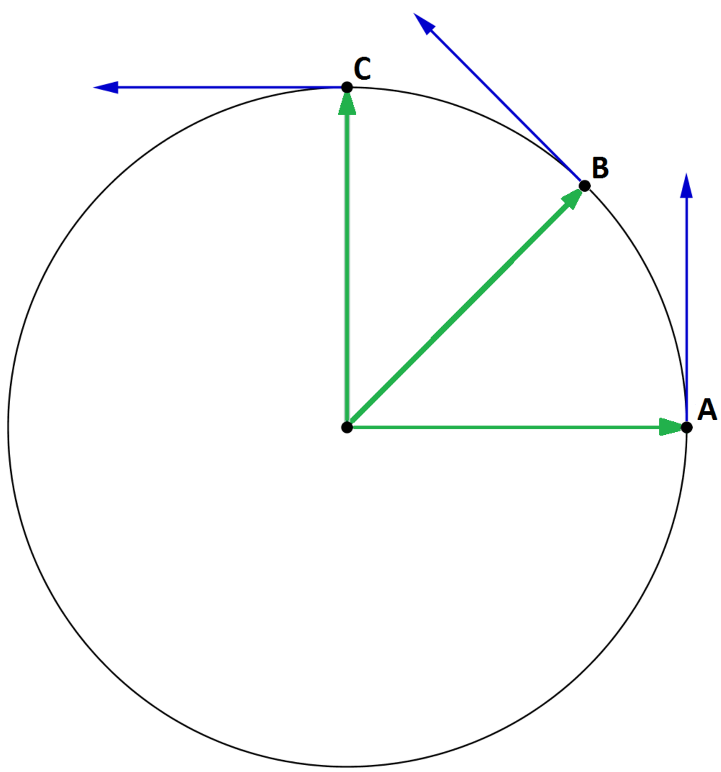
$a = \alpha r$

Centripetal Acceleration

When no other forces are acting, a moving object continues to move in a straight line. Therefore, if an object is moving in a circle there must be a force compelling it to do so. That force is directed to the center of the circle. The **Centripetal Force** is the force that causes an object to move in a circular path or curved path.

If the object is moving at a steady speed, you might think that there is no acceleration, and therefore no force. However, even though there is no linear or angular acceleration, circular motion does involve a change in velocity. The direction of the motion is changing. This kind of acceleration is called **centripetal acceleration**. When we start talking about direction, vectors become useful representations. First, we will use vectors to represent the position of the object as it moves along the circle. The position vectors will have a constant magnitude, which is the radius of the circle, r. The length of the circular path is the circumference of the circle, 2πr.

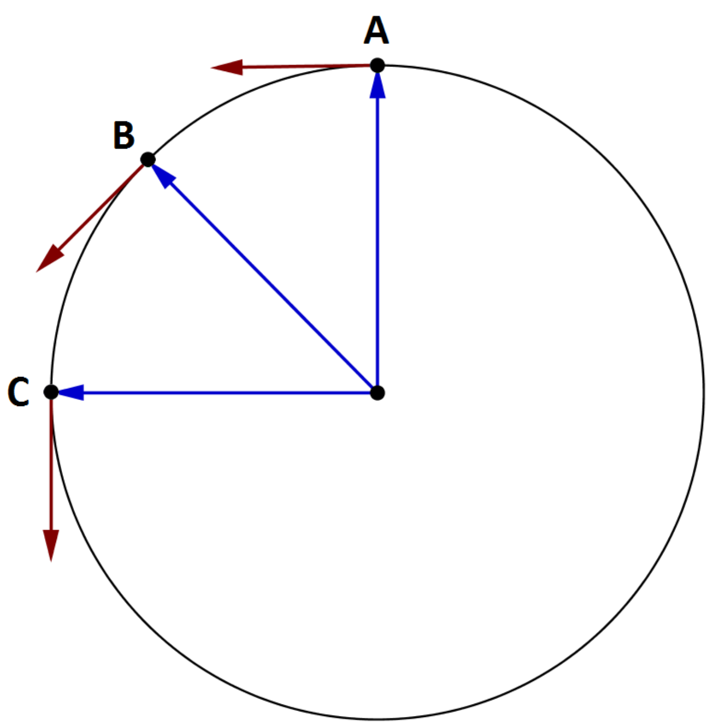
If the centripetal force were to suddenly disappear, the object would continue in a straight line. The direction of the velocity at any given instant is straight ahead, so the velocity vector is always tangent to the circle. That means that it is at a 90 degree angle to the position vector. The velocity vectors for an object moving counterclockwise are shown in blue in the image below:



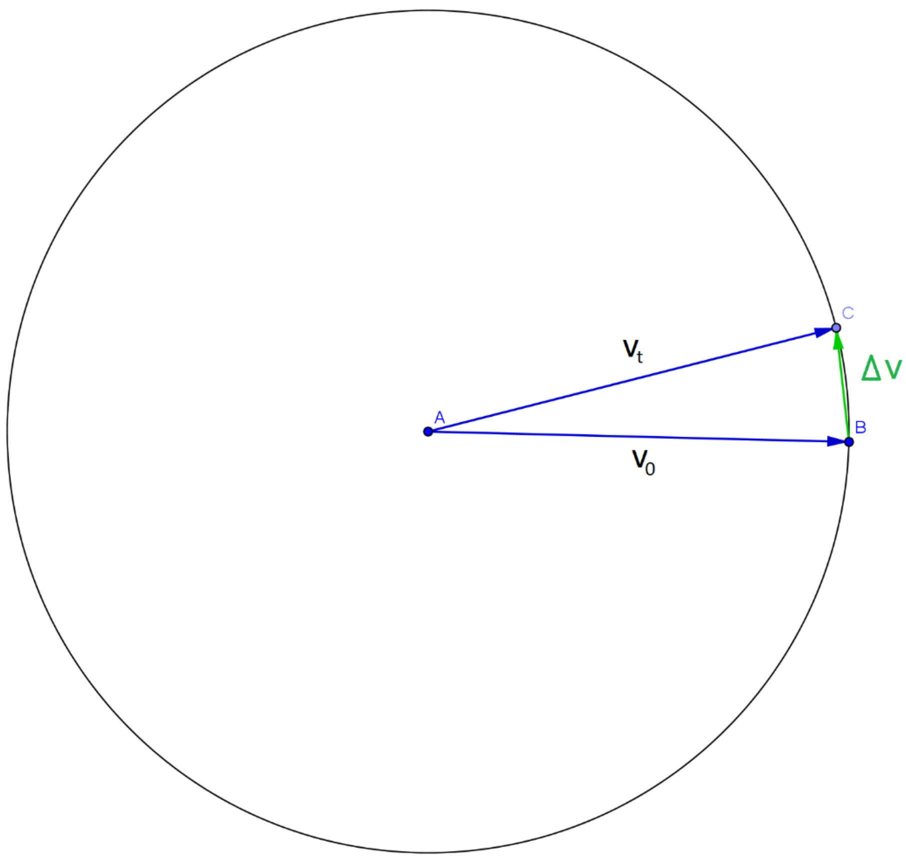
The magnitude of the velocity (the speed) will not change for the kind of circular motion we are looking at here. The length of the velocity vector remains constant. To calculate the speed, which is the magnitude of the velocity vector, we have to know how long it takes the object to go all the way around the circle. This length of time is the **period**, and it is usually indicated by a capital T. The speed is the distance, $2\pi r$, divided by the period T: $|\vec{v}| = \frac{2\pi r}{T}$. Here the absolute value sign indicates the magnitude of the vector v.

Note that in vector terms the velocity is the change in the position vector $\Delta|\vec{p}|$ divided by the elapsed time Δt. Since the position vector is changing constantly we would need $\Delta|\vec{p}|$ over a really tiny period of time to get an accurate measure. We are skipping over this problem by using the circumference of the circle as the sum of all of the really tiny vectors $\Delta|\vec{p}|$. That approach seems rather obvious here because the circumference of the circle clearly is the distance, but we will use the same method to measure the acceleration.

The acceleration is the change in the velocity. Normally the acceleration would be zero if the speed remains constant, but here we have to consider direction. There is only one force involved: the centripetal force. It is always directed toward the center of the circle. $\vec{F} = m\vec{a}$, and if the force is directed toward the center, the acceleration is also toward the center. That means that the acceleration vector is at a 90 degree angle to the velocity vector, and directly opposite to the position vector. The object is continually pulled toward the center, so the direction of the velocity changes constantly. We can determine the magnitude of the acceleration in the same way we determined the speed. You can take the velocity vectors and position them to form a new, smaller circle. The radius of that circle is the length of the velocity vector, the speed v. The circumference of the new circle is $2\pi v$. The acceleration vectors are at a 90 degree angle to the velocity vectors, which makes them tangent to the circle as you can see in the image below:



Acceleration is the change in the velocity over time. The period is still T, and the velocity vector rotates around the circle in one period. The total change in the velocity during this time is the distance around the circle, 2πv. That is really the sum of all of the tiny changes in the velocity vector. One such change is shown as Δv in the image below. When the changes are small enough they sum to the circumference of the circle.



The acceleration is the change in velocity over the elapsed time T: a = $\frac{2\pi v}{T}$.

Also, v = $\frac{2\pi r}{T}$ and we can substitute to get a = $\frac{2\pi}{T} \cdot \frac{2\pi r}{T} = \left(\frac{2\pi}{T}\right)^2$ r. That doesn't look exactly like the textbook formula a = $\frac{v^2}{r}$, but if you look carefully you can see that it is. If v = $\frac{2\pi r}{T}$, then v^2 = $\left(\frac{2\pi}{T}\right)^2 r^2$. If you divide that by r you get a, which is $\left(\frac{2\pi}{T}\right)^2$ r. Once you notice that, it makes more sense to substitute to eliminate T:

v = $\frac{2\pi r}{T}$, so T = $\frac{2\pi r}{v}$

a = $\left(\frac{2\pi}{T}\right)^2$ r

a = $\left(\frac{2\pi}{\frac{2\pi r}{v}}\right)^2$ r

a = $\left(2\pi \cdot \frac{v}{2\pi r}\right)^2$ r

a = $\left(\frac{v}{r}\right)^2$ r = $\frac{v^2}{r}$

Centripetal Acceleration: $a_c = \frac{v^2}{r}$

<u>This is how it looks in Calculus</u>:

A circle with a radius of 1 can be described by the equations x = cos t, y = sin t. If you set the position vector to be $\langle \cos t, \sin t \rangle$, the circle will be completed when t reaches 2π. To get a period of 1 we need to multiply by 2π so t can be smaller, and then divide by T to stretch things out.

For a circular path with radius r, and a period T, the position vector is $|\vec{p}| = \langle r\cos\left(\frac{2\pi t}{T}\right), r\sin\left(\frac{2\pi t}{T}\right)\rangle$.

The velocity vector is the derivative of the position vector. To get the derivative of a vector you take the derivative of its components. The period T is a constant, and we are taking the derivative with respect to the time t:
$\vec{v} = \langle -\frac{2\pi}{T} r\sin\left(\frac{2\pi t}{T}\right), \frac{2\pi}{T} r\cos\left(\frac{2\pi t}{T}\right)\rangle$.

From this you can see that the velocity vector is at a 90 degree angle, counterclockwise, to the position vector because $\langle -\sin t, \cos t\rangle$ is a 90 degree rotation of $\langle \cos t, \sin t\rangle$.

The velocity is the magnitude of the velocity vector: $|\vec{v}| = \sqrt{\left(\frac{2\pi r}{T}\right)^2\left(\sin^2\left(\frac{2\pi}{T}\right) + \cos^2\left(\frac{2\pi}{T}\right)\right)}$, *and since $\sin^2 x + \cos^2 x = 1$, the velocity is $\frac{2\pi}{T} r$. That makes sense, because it just says that the velocity is the distance around the circle divided by the period.*

The acceleration vector is the derivative of the velocity vector:
$\vec{a} = \langle -\left(\frac{2\pi}{T}\right)^2 r\cos\left(\frac{2\pi t}{T}\right), -\left(\frac{2\pi}{T}\right)^2 r\sin\left(\frac{2\pi t}{T}\right) \cos\rangle$. *The direction of the acceleration vector is directly opposite the position vector ($\langle -\cos t, -\sin t\rangle$ vs. $\langle \cos t, \sin t\rangle$).*

The magnitude of the acceleration is $|\vec{a}| = \sqrt{\left(\frac{2\pi}{T}\right)^4 r^2 \left(sin^2\left(\frac{2\pi}{T}\right) + cos^2\left(\frac{2\pi}{T}\right)\right)} = \left(\frac{2\pi}{T}\right)^2 r$.

If v is $\frac{2\pi}{T}$ *r, then* v^2 *is* $\left(\frac{2\pi}{T}\right)^2 r^2$*, and you can see that* $a = \frac{v^2}{r}$.

Centripetal force: $\mathbf{F_c = ma = m \cdot \frac{v^2}{r}}$

The centripetal force is the force that causes an object to deviate from its otherwise straight path to move in a circle. The centripetal force does not do work, since it is always perpendicular to the direction of the motion.

Look at the formula to see that the centripetal force must increase as the speed increases. If you swing a ball around on a string, you will notice that the faster the ball moves, the more force you must exert on the string.

The centripetal force can be less when the radius is larger. Curves on high-speed roads are designed to have a large "radius", so that the centripetal force required to keep your car on the road can be provided by the friction of your tires against the pavement.

Since the linear velocity v is equal to the angular velocity times the radius, we can also express the centripetal force in terms of angular velocity:

Centripetal force = $m \cdot \frac{(\omega r)^2}{r} = mr\omega^2$

For planets moving around a star, the centripetal force is equal to the gravitational force, $\frac{G \cdot mM}{r^2}$, where m is the mass of the planet and M is the mass of the star. In the section on Gravity, we stated that the value of the gravitational constant G is G is approximately 6.67384×10^{-11} m^3 kg^{-1} s^{-1}.

$$m\frac{v^2}{r} = \frac{G \cdot mM}{r^2}$$

$$v^2 = \frac{G \cdot M}{r}$$

Solving for M, we get M = $\frac{v^2 r}{G}$. This means that if we know the velocity and orbital radius of a planet with an unknown mass m, we can find the mass M of the star it is orbiting.

Roller Coasters

A roller coaster can go through a loop so that the cars are upside down, yet don't fall. When we look at the motion of these cars from a distance, we see that they are accelerating towards the center of the loop instead of proceeding in a straight line. There must be a net force directed toward the center of the loop. The magnitude of this net force is $F_c = m \cdot \frac{v^2}{r}$. There are two forces acting on a roller coaster car: gravity, and a normal force caused by the track pushing up or down on the car. At the bottom of the loop, the gravitational force vector points down, while the normal force is directed upward. Because the car is moving in a circle it is experiencing centripetal acceleration towards the center of the loop and **the two forces are not equal**. The net force is the normal force plus the gravitational force, which means that the normal force is larger than it would be if the car was standing still.

At the top of the loop, the normal force is pointing down. Together, the normal force and the gravitational force must add to the centripetal force. If the centripetal force is not large enough, there won't be a normal force and the car will lose contact with the track. The centripetal force $m \cdot \frac{v^2}{r}$ must be at least as large as the gravitational force mg, which can be accomplished by increasing the velocity and/or decreasing the radius. Many roller coasters have loops shaped such that the radius is smaller on the top than it is on the bottom. Increasing the velocity is not as simple as making sure that the car enters the loop with a particular velocity, because it will lose speed as it approaches the top of the loop. You must account for the loss of kinetic energy as the car gains height.

Example

A roller coaster on a frictionless track rolls down a steep incline and then enters a circular loop with a radius of 10 meters. Assuming the initial velocity is zero at the top of the incline, how far above the bottom of the loop does the top of the incline need to be?

Since the radius of the loop is already determined, we can say that the required centripetal force will be $m\frac{v^2}{10}$, which must be at least as large as the gravitational force mg. Notice that the problem doesn't specify a mass for the coaster cars, but it doesn't need to because m cancels out and we get $\frac{v^2}{10} \geq g$. v^2 refers to the velocity at the top of the loop, a full 20 meters above the bottom of the incline. Let's call the initial height h. The potential energy difference between h and the top of the loop is mg(h – 20). This is the amount of energy available for conversion to kinetic energy at the top: mg(h – 20) = ½ mv^2. m cancels out, so g(h – 20) = ½ v^2, or v^2 = 2g(h – 20).

$$\frac{v^2}{10} \geq g$$

$$2g(h - 20) \geq 10g$$

$$2(h - 20) \geq 10$$

$$2h - 40 \geq 10$$

$$h \geq 25 \text{ meters}$$

Personally, I wouldn't be getting on that roller coaster unless h is a lot higher than 25 meters.

<u>Practice</u>

1. Start with the same roller coaster at the top of a 25 m tall incline, and show that the radius of the loop must be equal to or less than 10 meters.

2. Show that the height of the incline is related to the radius of the loop by the formula $h \geq \frac{5}{2}r$.

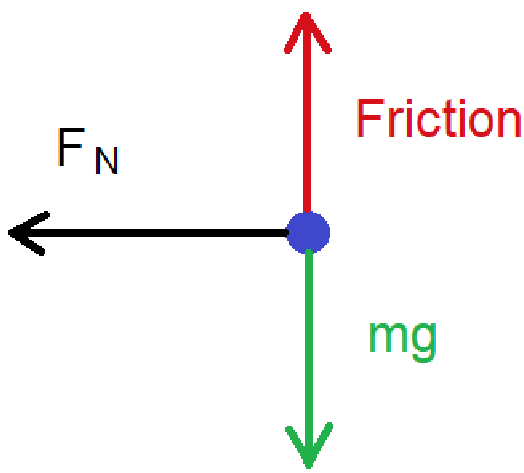
As a roller coaster car passes over a hill in the track, it is also experiencing centripetal acceleration. Again, there are only two active forces. The normal force pushes up on the car, and gravity pulls it down. The net force is equal to the centripetal force if the car stays on the track and actually moves around part of a circle. At the top of the hill the gravitational force mg must be at least equal to the centripetal force $m\frac{v^2}{r}$ required to accelerate the car around the hill. If the required centripetal force is greater than mg, the car will fly off the track. If the centripetal force is equal to mg, there is no normal force and the riders will feel weightless as the bottom of their seat no longer presses on them. To be safe, the centripetal force should be smaller than mg, so there will still be some normal force.

Just standing on the surface of Earth also causes you to experience centripetal acceleration. As the Earth spins, your centripetal acceleration is $\frac{v^2}{r}$. Part of the gravitational force (a rather small part) provides the centripetal force required for this motion. As a result, you actually experience a slightly smaller normal force than what you would expect.

Spinning Fair Ground Rides

Rides that spin people in a circle cause them to feel as if they are being pushed outwards. The fictitious force they feel is called the centrifugal force. That is not an actual force, but simply the tendency for a moving body to proceed in a straight line. Instead the people are redirected along a circular path by the normal force. In a Rotor or Gravitron ride, the floor drops down while the passengers are stuck to the wall. You may be asked to draw the forces involved, or explain why people don't slide down the wall.

Each passenger will still experience a gravitational force proportional to their weight, but since there is no vertical movement that force is balanced by something else. The centripetal force in this case is provided entirely by the normal force exerted by the wall of the ride. It has a magnitude of $m\frac{v^2}{r}$, directed horizontally towards the center of the ride. The normal force doesn't have a vertical component, but it does determine the magnitude of the friction between the passenger and the wall. The force of friction is directed straight up, opposing the force of gravity. Its magnitude is μF_{Normal}, where μ is the coefficient of static friction.

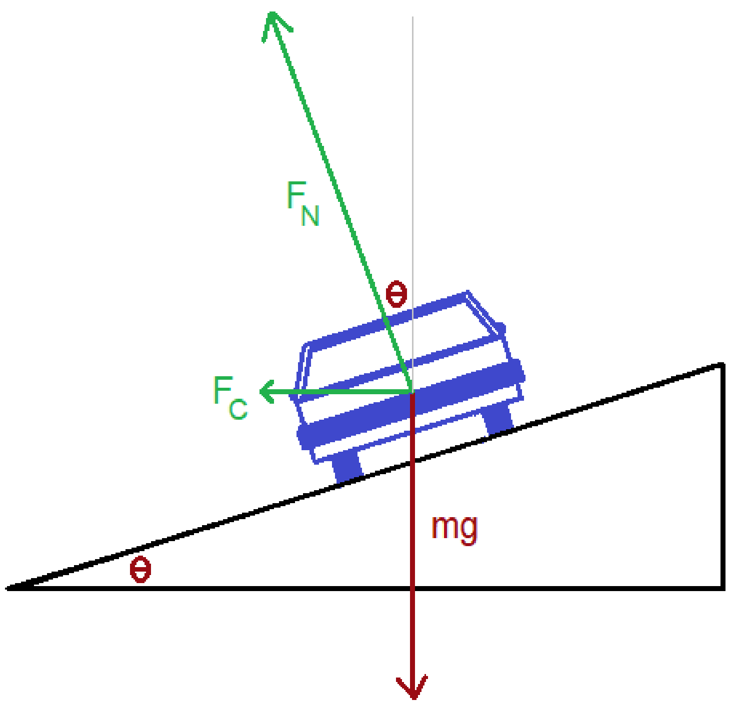


The force of friction is also proportional to the person's weight: $F_{friction} = \mu\, m\frac{v^2}{r}$. As a result, a heavier person is no more likely to slide down the wall than a lighter person. The only difference between passengers could be μ, the coefficient of static friction determined by their clothing.

Banked Roads

Since friction is often not sufficient to allow a car to navigate a tight curve at a reasonably high speed, the road may be banked (tilted). When the car is tilted at an angle, the normal force is still perpendicular to the road. We can calculate the magnitude of this normal force, and then find the component of it that is horizontal, pointing to the center of the circle. This component provides centripetal force. Gravity as always points straight down, so it has no horizontal component. It does not contribute anything to the centripetal force.

When we looked at ramps earlier, we saw that the component vector of gravity perpendicular to the ramp's surface has a magnitude of mg cos θ. This resulted in a normal force with the same magnitude, pointing up. Things are different here because the normal force is also going to supply the centripetal acceleration $\frac{mv^2}{r}$. It will be larger due to the velocity of the car.



Here we will not consider friction, so you'll have to imagine that the road is covered by a sheet of ice. If the car is going too fast, it will gain altitude as it starts to leave the road on the higher side. If the car stands still or travels very slowly, it will slide down the incline because there is no friction. For our calculations we will assume that the speed of the car is exactly the ideal speed that will cause it to round the curve without going higher or lower on the incline. Since there will be no vertical movement, we know that the vertical forces are balanced:

$F_N \cos \theta = mg$.

The centripetal force F_c, with a magnitude of $\frac{mv^2}{r}$, is equal to $F_N \sin \theta$.

Now we have 2 useful expressions for F_N: $F_N = \frac{mg}{\cos\theta}$ and $F_N = \frac{mv^2}{r\sin\theta}$. This allows us to create a formula that determines the angle required to accommodate a particular velocity for a curve with a given radius, or the ideal velocity for a particular angle.

$$\frac{mg}{\cos\theta} = \frac{mv^2}{r\sin\theta}$$

$$\frac{g}{\cos\theta} = \frac{v^2}{r\sin\theta}$$ Cross-multiply:

$$g\, r \sin\theta = v^2 \cos\theta$$

$$v^2 = \frac{g\, r\sin\theta}{\cos\theta}$$

$$\mathbf{v^2 = g\, r \tan\theta}$$

Note that mass has canceled out, so this formula works for all kinds of vehicles.

Satellites

For satellites orbiting the earth, the orbit is roughly spherical. The satellite is constantly falling (accelerating) towards earth, which is why astronauts in orbit are weightless. To find the velocity of a satellite, consider that the only force acting on it is that of gravity, and this force causes it to experience centripetal acceleration. Force is equal to mass times acceleration, and since the centripetal acceleration a is $\frac{v^2}{r}$, the force acting on the object to produce this acceleration is $m\frac{v^2}{r}$. For a satellite, the force is provided by gravity, so it is $\frac{G\,m\,M}{r^2}$.

$$\frac{G\,m\,M}{r^2} = m\frac{v^2}{r}$$

$$\frac{G\,m\,M}{r} = m\,v^2$$

The smaller mass m cancels out, allowing us to find the speed v from the equation $\mathbf{v = \sqrt{\frac{G\,M}{r}}}$.

The period of the satellite, T, is then found by taking the distance divided by the speed: $\mathbf{T = \frac{2\pi r}{v}}$. Remember that the period T will be in seconds if you are using standard units, so divide by 3600 to convert to hours.

Example

A person standing at the equator is actually moving in a circle at about 464 m/sec as a result of the Earth's spin. At this point the radius of the Earth is about 6,378,000 meters. What is the reduction in the normal force experienced by a 65 kg person?

The force of gravity is the only force acting here. Its magnitude is 65 kg times 9.8 m/sec^2 or 637 N. Part of the gravitational force is used to provide the centripetal force, while the rest is felt by the person as the normal force. Here the magnitude of the centripetal force, m $\frac{v^2}{r}$, is $\frac{65 \cdot 464^2}{6{,}378{,}000}$ or about 2.19 N. To convert this to a weight reduction in kg, use m = $\frac{F}{g}$. If a 65 kg person stands at the equator they would feel about 0.22 kg lighter than they would if they were standing at the North Pole.

Torque

If you want to open a door, you push or pull on it in a direction perpendicular to the door. The door is easiest to move if you push on it further away from the hinges – just try it. It makes sense to put the handle of a door as far away from the hinges as possible. To open the door you can push it gently over a long distance near the handle, or over a shorter distance near the hinges by using a greater force.

When you consider circular motion, it is not so helpful to just think in terms of force, because it depends where you apply that force. Instead, we use **Torque** (τ) to measure the force of rotation. The torque is the product of the force and the distance to the pivot point. So, if you apply a force of 1 Newton to a door 50 cm away from the hinge, you are using a Torque of 1N x 0.5m = 0.5 N-meters or 0.5 Joules. When using a wrench to turn a bolt, the distance will be the length of the wrench. Of course that comes with a formula:

$\tau = F\ r$ where r is the distance to the pivot point, which is the radius of the rotation. The pivot point is sometimes called a **fulcrum**.

If you are trying to open a door, you would normally push or pull on it to apply the force at a 90 degree angle. You do so instinctively because that is the most efficient use of your muscle strength. Try using a very different angle to see what happens. The force F in the formula is the component of the force that is applied at a 90 degree angle (perpendicular to the line that measures the distance). To make things more challenging textbook problems often use an angle other than 90 degrees. When a force is applied at an angle rather than perpendicular to r, find the component of the force that actually is perpendicular, and use that to calculate the torque.

If the rotation resulting from the applied force is counterclockwise, we say that the torque is positive (it unscrews a bolt), and if the rotation is clockwise we say that the torque is negative. To remember this you can relate it to angles – as you move counterclockwise from the x-axis you get a positive angle, and clockwise from the x-axis the angle is negative.

When two objects are balanced on a fulcrum (like a seesaw), their torques are equal and in opposite direction so that the net torque is 0. The torque is equal to the weight of each object times its distance from the fulcrum.

Rotating objects follow the laws of physics just like objects that move in a straight line. If something is rotating, it keeps rotating unless a force like friction stops it. When something is rotating at a steady speed, the angular acceleration is zero. A force is required to change the speed, and that force is applied as a torque. Friction can act as a torque.

The work done by a torque is still Force times distance, but we can convert that to rotational terms. The distance over which the force acts in circular motion is the arclength S, which is equal to $r\theta$. So, Force x distance = F $r\theta$. The torque is Fr, which means we can express the work done by a torque as $\tau\theta$.

Work = Fd = $\tau\theta$

Rotational Inertia

For regular linear motion F = ma, so the magnitude of the acceleration caused by a force depends strictly on the mass. In this situation an object acts as if all of its mass were located at a single point, because all of the parts of it move with the same velocity. For a rotating object the acceleration also depends on the mass, but things are a bit more complex. Imagine a 3 kg weight located on the outer edge of a 10 meter spinning disk. As the disk rotates once, the weight travels all the way around the circumference – a distance of 2π times 10 meters. Now consider a second 3 kg weight located 5 meters from the center. This weight only travels half the distance of the first, or 2π times 5 meters for each revolution. Although the angular velocity is the same for both weights, the weight at the edge of the disc is moving twice as fast. The edge weight has more impact in terms of how hard it is to get the disk spinning or to get it to stop.

We can describe the both the weight and the impact of it by the term **rotational inertia** (also called moment of inertia). The rotational inertia, **I**, is the resistance of an object to changes in its rotation due to a torque. Rotating objects do not act as if all of their mass were located at a single point. Instead, the resistance to a torque depends on how far from the center each bit of the mass is. If most of the mass is closer to the center it is easier to get the object to rotate.

An object's rotational inertia depends on its mass, and on the distance of the mass from the center of rotation, which is r. It is actually r that has the most impact. You can see that if you consider the kinetic energy of the two masses described above. To simplify things, we will imagine that they are heavy but extremely tiny. Kinetic energy is ½ mv^2, so let's translate that into rotational motion by using the angular velocity ω. v = r ω, which means that each mass has a kinetic energy of ½ m $(r\,\omega)^2$, which is ½ m $r^2\,\omega^2$. The quantity mr^2 shows the impact of both the mass and its distance from the center of rotation.

Rotational Inertia: **$I = mr^2$**

The units of rotational inertia are kg m^2. The actual rotational inertia depends on the shape and density of the rotating object. For a single mass, you would find I by taking each particle of the mass and multiplying it by the square of its distance from the axis, and then taking the sum. That sounds like a lot of work, but calculus can make it easier.

By analogy with Newton's second law, F = m a, we can say that

Torque = $I\,\alpha$

This is just a straight translation of Newton's law: $\tau = F\,r$, $I = mr^2$, and $\alpha = \frac{a}{r}$. The torque is equal to the rotational inertia times the angular acceleration. Recall that the angular acceleration is the change in the angular velocity.

Angular Momentum

The formula for **angular momentum** is analogous to that for regular momentum, m v.

$L = I\,\omega$

Just like regular momentum, angular momentum is conserved. This is amazing, and also important to know for exams. For regular momentum, the mass m is always the same. However, rotational inertia is dependent on the radius, so if some of the mass changes position during rotation, the angular velocity may change. The example usually given is a spinning skater moving his or her arms in or out to control the speed of rotation. If more of the mass is located closer to the center of rotation it becomes easier to move. The *effective* mass, I, decreases so that the angular velocity, **ω**, has to increase. The effect is dramatic because I depends on the square of the radius. For collisions involving circular motion, you can set the total angular momentum before the collision equal to the total angular momentum after the collision.

Conservation of angular momentum is also seen in the solar system. Planets and moons do not orbit in perfect circles. These orbits are elliptical, and the speed of the orbiting body is not constant. Where the radius of the orbit is smaller the speed is higher, and then the body slows down as the radius increases. The angular momentum stays constant because there is no external torque on the system.

The conservation of angular momentum causes the Coriolis effect. The Coriolis effect makes storms in the Northern hemisphere spin counter-clockwise, while storms in the Southern hemisphere spin clockwise. As the Earth rotates on its axis, points closer to the equator move faster than points near the pole. When air tries to move from the higher pressure edge of the storm to the low pressure center, its momentum is conserved, and it ends up traveling in an arc instead of in a straight line.

Equations for circular motion that are analogous to those for linear motion:

linear: $\mathbf{d = v_i t + \frac{1}{2} a t^2}$ circular: $\mathbf{\Delta\theta = \omega_i t + \frac{1}{2} \alpha t^2}$

linear: $\mathbf{v = v_i + a t}$ circular: $\mathbf{\omega = \omega_i + \alpha t}$

linear: $\mathbf{d = \frac{v_f^2 - v_i^2}{2a}}$ circular: $\mathbf{\Delta\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha}}$

This similarity is not arbitrary; it is simply the result of the relationships that we looked at before. The displacement d is the change in the arclength rθ, the velocity v is rω, and the acceleration a is rα. Translating the equation for displacement we get:

$d = v_i t + \frac{1}{2} a t^2$

$\Delta r\theta = r\omega_i t + \frac{1}{2} r\alpha t^2$

$\Delta\theta = \omega_i t + \frac{1}{2} \alpha t^2$

Things work the same for all of the equations. For $d = \frac{v_f^2 - v_i^2}{2a}$ we get

$d \cdot 2a = v_f^2 - v_i^2$

$\Delta r\theta \cdot 2r\alpha = (r\omega_f)^2 - (r\omega_f)^2$

$\Delta\theta \cdot 2r^2\alpha = r^2\omega_f^2 - r^2\omega_f^2$

$\Delta\theta \cdot 2\alpha = \omega_f^2 - \omega_f^2$

$\Delta\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha}$

Ramps

You may not have noticed, but when you were doing ramp problems they always involved a block sliding down a ramp, rather than a ball rolling down. That is because the situation is more complex when an object rotates as it moves. As we saw earlier, rotating objects have rotational kinetic energy, which is given by ½ m r^2 ω^2, or ½ I ω^2.

Rotational KE = ½ I ω^2

When a ball rolls down a ramp, some of the work done by gravity translates into linear movement, and some of it is used to provide the rotation. This means that a rolling ball actually takes longer to get to the bottom of the ramp.

If you need to find the details of that, it's not going to be as simple as taking the mass of the ball and its radius. Each separate bit of mass and its radius needs to be accounted for. Luckily there is a formula for this for various shapes, and for a solid ball the rotational inertia is $\frac{2}{5}$ mr^2. Note that a hollow ball of the same mass has a higher rotational inertia because all of the mass is near the outside. A hollow ball will take longer to roll down a ramp than a solid ball with the same mass because gravity has to do more work to get it to rotate. Balls can also roll up a ramp if they have sufficient kinetic energy. Where the ball stops, at the highest point, all of the linear kinetic energy and rotational kinetic energy have been converted to potential energy.

We are so used to the idea that a ball will roll downhill rather than slide that we don't really think about why it does that. When gravity tries to move the ball down the hill, it is opposed by friction. The friction that keeps the ball from starting to move is static friction. This is the force that provides the torque that gets the ball rotating. If the ball doesn't slip, static friction continues to provide a torque. Otherwise the point of contact between the ball and the ramp actually moves along the surface and the ball will experience a torque due to kinetic friction, which is smaller.

Note that it is theoretically possible to move a sphere without getting it to rotate. We could push a ball along a frictionless surface. To start that motion, we would need to apply a force to the sphere without creating a torque. This means the force must be perfectly aligned with the center of mass of the ball, to push it without making it turn.

Problems involving balls rolling down frictionless ramps can involve tracks with interesting shapes, often including vertical semicircles. The centripetal force does no work, because it is always perpendicular to the motion, so it will not change the speed. It only changes the direction of the velocity. However, a ball will speed up along a steeper down section of track, and slow down when it goes up a hill, simply because it temporarily encounters a larger or smaller component of the gravitational force. The shape of a track will affect the average velocity. The final velocity is determined by the net change in height, and the moment of inertia of the ball.

MACHINES

A machine is a device with which you can do work in a way that is easier or more effective.

A machine makes work easier by changing the amount of force you exert, the distance over which you exert your force, or the direction in which you exert your force.

The input force is the force exerted by the person on the machine (also called the **effort force**). The force exerted by the machine is the output force (**resistance force**).

Mechanical advantage = $\frac{\textbf{Output Force}}{\textbf{Input Force}}$

In reality, some of your work is lost to friction as you use a machine. This is expressed as the efficiency of a machine.

Efficiency = $\frac{\textbf{Output Work}}{\textbf{Input Work}}$ **x 100%**

You can improve the efficiency of a machine by keeping it clean and well lubricated.

These are the basic types of machines:

1. The lever (1st, 2nd and 3rd class)

Since Work is equal to Force times distance, we can do the same amount of work by exerting a large force over a short distance, or a smaller force over a longer distance. If you only have so much muscle strength available, you may find it easier to go with a longer distance. A lever can help you lift an object that would otherwise be too heavy for you to move, at the expense of you moving the end of the lever much further than how far you actually displace the object.

To understand levers better you can think in terms of torque. A lever rotates around a pivot point, and this rotation is caused by a torque (usually you pushing down on the lever). For a simple 1st class lever like a crowbar the shorter end rotates up as the longer end rotates down. Torque is the perpendicular force times the distance from the pivot point, so force times distance is an equal quantity for both ends of the lever. Where the distance is shorter the force is proportionally greater.

A wheelbarrow is an example of a second class lever. The load is located between the pivot point and the input force. As you lift the wheelbarrow by its handles, it rotates around the pivot point, which is the wheel. The handles rotate a lot, but the center of gravity of the load doesn't move so far because it is much closer to the pivot point. If you are doing calculations remember that force times distance for the handles is equal to force times distance for the center of gravity of the load.

For a third class lever, the input force and the load are on the same side of the pivot, but you will apply your input force close to the pivot while the center of gravity of the load is further away. An example of this is your forearm which pivots at the elbow. Due to anatomical constraints, the input force applied by your biceps is very close to the elbow joint. The biceps attaches near the elbow, on the same end of it as your hand. As the muscle contracts over a short distance, your hand rotates much further. There is no mechanical advantage, but because the load moves more distance than the muscle over the same time there is a speed advantage.

2. The inclined plane

An inclined plane allows you to lift a heavy load by applying a smaller force over a longer distance. The work done is the same as if you lifted the load straight up. The mechanical advantage is the length of the ramp divided by the height.

3. The wedge

A wedge converts force applied to the blunt end into forces normal to its sloped surfaces. This allows for the splitting or lifting of an object. Axes, knives, and nails are wedges, and so are your front teeth. The mechanical advantage is the length of the wedge divided by the width. A longer wedge requires less input force.

4. The screw

A screw converts rotational motion into linear movement. The screw turns quite a bit for a small forward gain, especially if the threads are close together. This gives it a large mechanical advantage, but that ideal advantage is often reduced by a fair amount of friction. A good way to use the screw principle is to move water. Screw pumps can lift large amounts of water to a higher elevation with minimal input force, and without harming fish.

5. The wheel and axle

The wheel and axle can supply a mechanical advantage due to the fact that the axle, with its smaller radius, rotates over a shorter distance than the wheel. This allows for a smaller input force when the wheel is rotated. A screwdriver is an example of a wheel and axle, because the handle rotates over a greater distance than the shaft, allowing you to apply more force on a screw. Calculate the mechanical advantage by dividing the radius of the wheel by the radius of the axle.

6. The Pulley

A single pulley only changes the direction of the applied force. However, by using two pulleys we can essentially lift a weight with two sections of rope. This cuts the input force in half, but because you are using two sections of the same rope you have to move it twice as far. The ideal mechanical advantage is 2 in this case.

TENSION

Tension is a force transmitted by a rope (or string or chain) that is pulled tight by forces acting on both ends.

While it may seem straightforward to calculate the tension in a rope, there are some complications. First, consider a 10 kg mass suspended from a ceiling by a 1 meter long rope. The mass experiences a downward force of 98 N, which is counteracted by the tension in the rope. Since the mass is not moving, we conclude that the tension in the rope is 98 N, directed upwards.

One complication to this would be the mass of the rope. When you look at the middle of the rope, you can see that the top half is holding up a 10 kg mass plus the weight of the bottom half of the rope. The tension force here has to be more than 98 N. Because of this, ropes or strings in physics problems conveniently have no mass. Just think of that mass as being insignificant.

Another problem occurs when you try to look at the forces at both ends of the rope. The tension force is the same everywhere along the rope, and it is felt at both ends. If the tension in a rope is 10 N, there will be a 10 N force at both ends, directed toward the opposite end. Do not add the tension forces at each end! There is only one tension, although there are action and reaction forces operating according to Newton’s Third Law.

It helps to know that the tension is always constant throughout an idealized rope unless there is an additional force acting somewhere along the length of the rope.

Pulling Multiple Blocks

The figure below shows two blocks connected by a rope (A), and being pulled along a frictionless surface by a second rope (B).

Your question may ask where the tension is greater, at point A or point B. If you just think about it, you will realize that the rope at A only has to pull the 10 kg weight, while the rope at B is pulling both weights. The second rope has more tension.

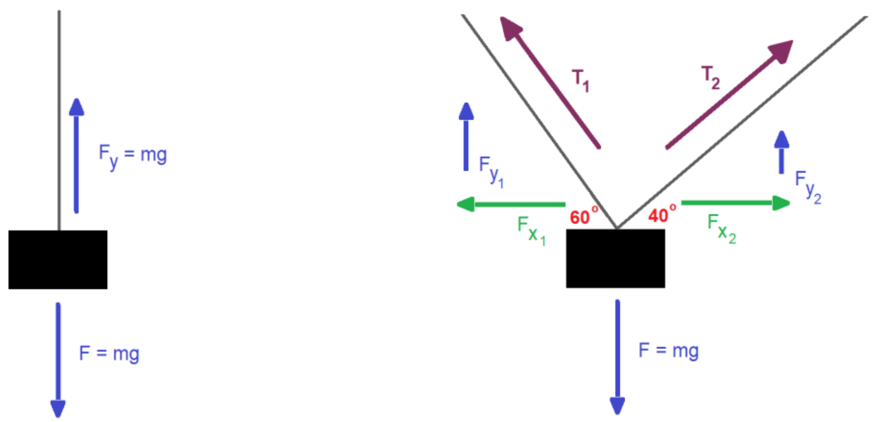
If the force pulling both blocks is 45 N, calculate the tension at points A and B.

At point B, we can see the two blocks as a single interconnected mass of 15 kg. The force pulling on this mass is 45 N, so the tension at point B is 45 N. The applied force is causing the combined mass to accelerate. F = ma, so 45 = 15 x a. The blocks are accelerating at 3 m/sec^2. To find the tension at point A, we have to consider the 10 kg mass separately. It is accelerating at 3 m/sec^2, and since its mass is 10 kg it must be experiencing a force of 30 N. The only force is the tension force at A, so that is 30 N. To check your work, you can consider the 5 kg mass separately. The tension in rope A is equal in magnitude at both ends, although opposite in direction. That means that the 5 kg mass is experiencing two forces: a tension force of 30 N to the left, and a tension force of 45 N to the right. The net force on this block is 15 N, to the right, which will cause it to accelerate at 3 m/sec^2, as expected.

An Object Suspended From Two Strings

The first thing to consider is why you might want to hang an object from two strings instead of one. Two strings can take a bigger load than one, so if that is the concern we can use two strings straight up and down. Now the force of gravity that pulls the object down in the y-direction is balanced by the two forces of tension provided by the strings in the upward y-direction.

Another consideration is that an object on a single string might swing around a lot. To reduce that swing, you can hang your two strings further apart. If you draw the forces provided by the string as vectors, you can see that there is a horizontal (x) component as well as a y-component. The horizontal components of the force from each string keep the object from moving much in a side-to-side direction. When the object is not moving, these two x-component forces are balanced – they are equal and opposite. The y-components of the string forces balance the force of gravity that is pulling the object down. The potential drawback here is that there is now more tension in your strings, since the y-direction forces haven't changed, so you may need stronger strings.



Anyway, solve problems of this type by drawing the string tension forces as vectors. Find the x and y-components of these vectors. Also determine the force of gravity on the object: F = ma, or mg in this case. And yes, you use acceleration here even though the mass is not moving. Think of it as potential acceleration.

On the right in the image above, a 1.0 kg mass is suspended from two strings angled at 50 and 40 degrees. The force of gravity on the mass is 1.0 x 9.8 or 9.8 N. The upward forces exerted by the strings must add up to 9.8 N: $Fy_1 + Fy_2 = 9.8$. These upward forces are the y-components of the string tension forces, but we don't know the magnitude of these tension forces (T_1 and T_2). Not to worry though, there is an important clue in the fact that the mass is not moving. That means that the horizontal components Fx_1 and Fx_2 must be equal. $Fx_1 = \cos 60^o\, T_1$, and $Fx_2 = \cos 40^o\, T_2$:

$\cos 60^o\, T_1 = \cos 40^o\, T_2$

Also, $Fy_1 = \sin 60^o\, T_1$ and $Fy_2 = \sin 40^o\, T_2$, and these two component forces add to 9.8N:

$T_1 \sin 60^o + T_2 \sin 40^o = 9.8$

This gives you a system of equations that allows you to determine the magnitude of both T_1 and T_2. Using rounded values for the sine and cosine:

$0.5\ T_1 = 0.766\ T_2$
$0.866\ T_1 + 0.643\ T_2 = 9.8$

Solve this system by substitution:

$T_1 = \frac{0.766}{0.5} T_2$

$T_1 = 1.532\ T_2$

$0.866\ (1.532\ T_2) + 0.643\ T_2 = 9.8$

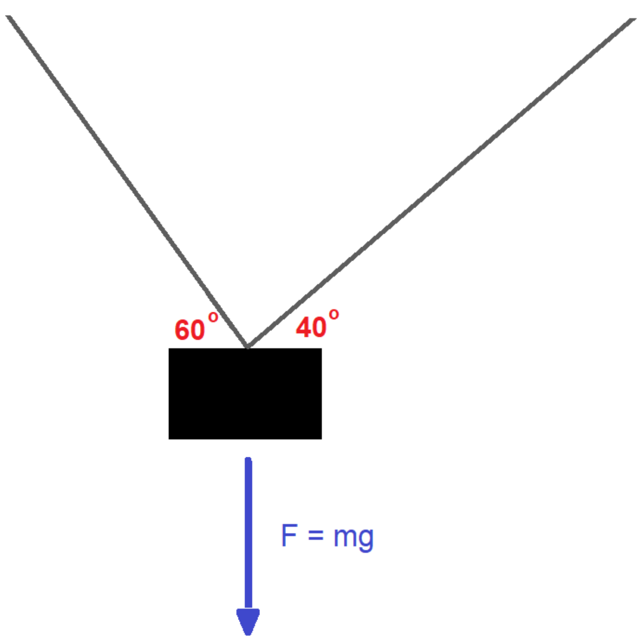
$1.327\ T_2 + 0.643\ T_2 = 9.8$

$1.970\ T_2 = 9.8$

$T_2 = 4.975$, which means that $T_1 = 7.621$. Round to two significant figures:

$T_1 = 7.6$ N and $T_2 = 5.0$ N

Look at the picture again:

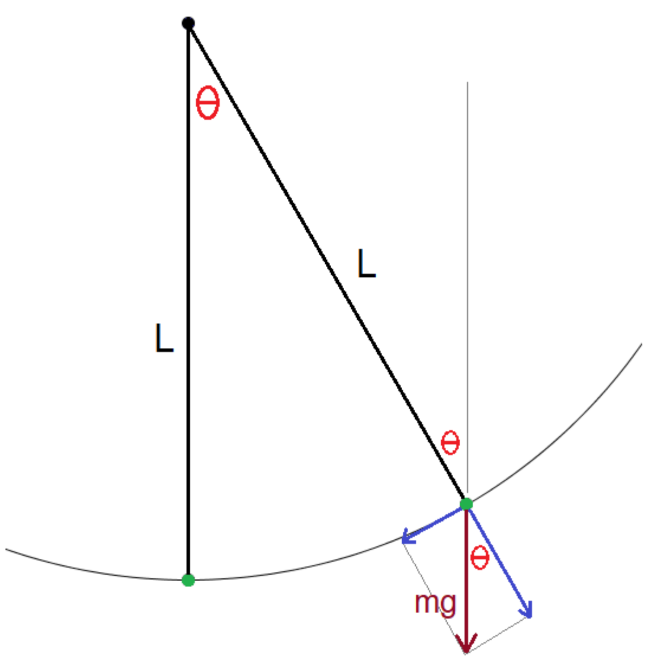


Now we know that there is a fair bit more tension in the string on the left. That kind of makes sense because the left string looks like it is holding up more of the weight of the block. If that left string is also shorter as shown, it is more likely to break than the one on the right.

The picture doesn't show much detail, so you can easily imagine that there is just a single long string passing through a ring attached to the block. Although the tension in a single string is usually the same throughout, in this case there are not just forces at the ends of the string. The weight of the block is causing a force somewhere along the length of the string. The tension is different in each part of the string.

Pendulum Tension

Gravity is often the first thing we think of for pendulum forces, but the most important fact is that the mass is swinging in an arc so that it is undergoing circular motion. That means that there is a net force causing it to deviate from a straight path. That net force is the centripetal force $F_c = m\frac{v^2}{r}$. The radius of the arc is equal to the length of the string L. The net force will have two components: gravity and the tension in the string.



The force of gravity parallel to the string, opposing its tension, is mg cos θ. (Notice that we can mark several angles as θ because they are either alternate interior angles or vertical angles.) The centripetal force is the sum of the tension in the string, T, and the component of the gravitational force:

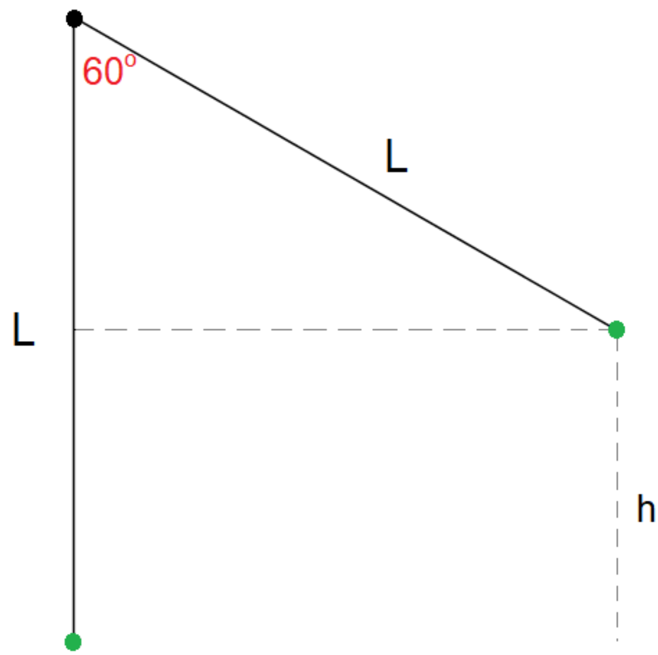
$m\frac{v^2}{r}$ = T + mg cos θ. Here you will want to assign some directions: T should be positive and mg cos θ should be negative. When the mass is at its highest position, the velocity is zero and T = - mg cos θ. As the mass moves to its lowest position it gains speed, so the centripetal force must increase. At the same time the angle θ decreases, and more of the gravitational force pulls against the string. The tension in the string increases. At the bottom of the swing, gravity pulls down with a force of mg, so the string must pull up with a larger force than that. That force must have an excess magnitude sufficient to provide a centripetal force of $m\frac{v^2}{r}$.

<u>Example</u>

A 20 g mass is attached to a string of length L. The pendulum is pulled out until the angle θ is 60^o and then released. What is the tension in the string at the moment the pendulum is released? What is the tension in the string when the pendulum reaches the lowest point of its swing?

When the pendulum is released its initial velocity is zero. The only force creating tension in the string is the component of the gravitational force, mg cos θ. T = (0.02)(9.8)(0.5) = 0.098N.

At the bottom of the swing, the mass has picked up a lot of velocity. All of the gravitational potential energy has been converted to kinetic energy. But what is that potential energy? To know that we must find the initial height h as shown in the diagram below.



In this picture you can see that when the string is vertical, its length L can be divided into two parts. The top part is L cos 60°, while the bottom part is h. We can say that h = L – L cos 60°. Since cos 60° is just 0.5, we have h = 0.5 L. The potential energy is mgh, which is converted to kinetic energy: ½ mv^2. m cancels out, and gh = ½ v^2. (9.8)(0.5L) = (0.5)v^2, so v^2 = 9.8L. The centripetal force has to be m $\frac{v^2}{r}$, or (0.02) $\frac{9.8L}{L}$ = 0.196. At the bottom of the swing, the gravitational force is (.02)(9.8) = 0.196N, pulling down. To get a net force of 0.196N pulling up, we need a tension of 0.196 + 0.196 = 0.392N.

Pulley Problems

A pulley consists of a wheel with a groove in it that allows a rope to move easily around the bend created by the pulley. The wheel spins as the rope goes through, which reduces friction. Problems of this type that you are likely to encounter will involve single frictionless pulleys, so the force of friction doesn't slow things down. The pulleys in physics problems also typically have no mass. Why do we need a massless pulley? Well, the way pulleys work is that the wheel inside rotates. There is now more moving mass in the system, and if the net force stays the same the acceleration decreases. Since you have to calculate that acceleration exactly, a massless pulley is more convenient. The AP exam assumes that you know that the actual mass of the pulley reduces the acceleration.

Always keep in mind that all an *ideal* pulley does is change the direction of the force. If your problem seems confusing you can simply eliminate the change in direction from the equation and consider the rope as if it was in a straight line. The longitudinal forces on the rope are applied at the ends, and the pulley creates a normal force only directly perpendicular to the rope. This allows the tension to remain the same on both sides of the pulley, and throughout the rope. However, if there was any friction or mass related to the pulley that would count as a force along the rope, which could cause tension to be different on either side of the pulley.

Example 1

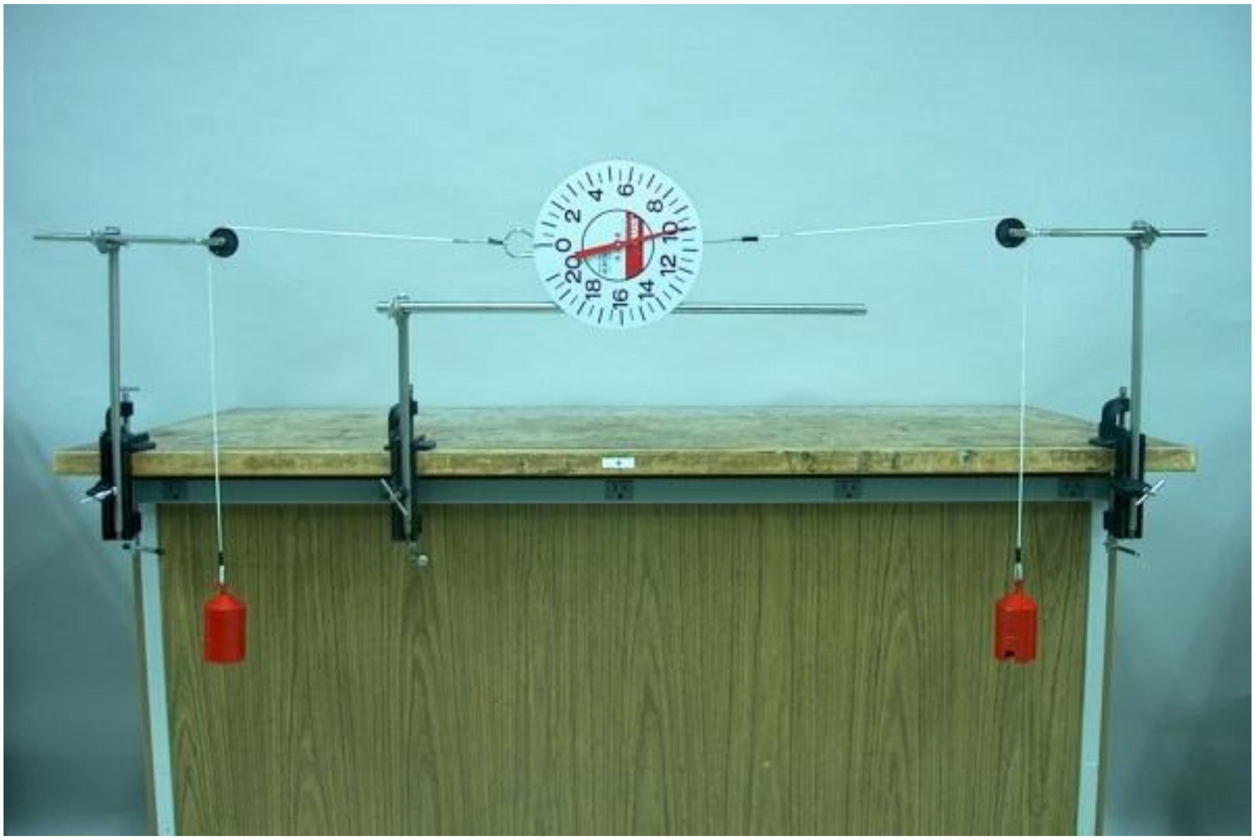
Two 10 kg masses are suspended from the ends of a rope that passes through a single massless frictionless pulley hanging from a ceiling. What is the tension in the rope?

At one end of the rope, there is a force of 10 x 9.8 or 98 N pulling down on the rope. The same goes for the other end. The forces are balanced, so the rope doesn't move. It may surprise you that the tension in the rope is 98 N rather than 196 N. The tension is the same everywhere along the rope, so you can just calculate it at one end. The force of gravity on one of the masses is 98 N, which is balanced by the tension in the rope. This situation is not really different from when we looked at a 98 N mass suspended from the ceiling. We said that the tension in the rope was 98 N, and it seemed straightforward because we only looked at the mass and the attached rope. At the other end, the tension in the rope pulled down on the ceiling, and the ceiling pulled back with a force of 98 N according to Newton's Third Law. In this example, the ceiling has effectively been replaced with a 10 kg mass that is doing the same job.

Note that if you suspend the pulley by a rope instead of attaching it directly to the ceiling, the tension in that top rope will be 196 N.

Check out this link, which shows two 1 kg masses suspended from a string:

http://web.physics.ucsb.edu/~lecturedemonstrations/Composer/Pages/12.39.html



Example 2

Using the same setup as in example 1, we replace one of the 10 kg masses with a 4 kg mass. The 10 kg mass starts to fall, pulling the 4 kg mass upwards. What is the acceleration of the 4 kg mass? What is the tension in the rope?

The question is phrased to disguise the fact that both masses are accelerating at the same rate. Because the masses are connected by a taut rope, they act as a single mass. That means we need the net force on the system, and the combined mass. F = ma. The net force is the difference between the downward pull of gravity on the 10 kg mass and the downward pull on the 4 kg mass. 10 x 9.8 N – 4 x 9.8 N = 58.8 N. This force acts on a total mass of 14 kg: 58.8 = 14 x a. The acceleration a is 4.2 m/sec^2. The direction is of that acceleration is downward for the larger mass, and upward for the smaller mass.

That part was simple because the connected masses are a single object as they accelerate. To find the tension however, we need to look inside of that object and consider the parts separately. Let's first look at the 10 kg mass. It is being pulled down by gravity with a force of 98 N, but the tension in the rope is slowing it down. We don't know that tension, but not to worry, we have the acceleration! If a 10 kg mass is accelerating at 4.2 m/sec^2, the net force on it must be 42 N.

$F_{net} = F_{gravity}$ – Tension.

42 N = 98 N – Tension

Tension = 56 N

To check that if this is correct, we can see if it works out the same at the other end of the rope, since the tension is constant everywhere when we use an ideal pulley. The 4 kg mass experiences a downward force of 4 x 9.8 = 39.2 N due to gravity. However, it is accelerating upward because the tension force in the rope is greater than that. The acceleration is 4.2 m/sec^2 upward, so the net force, mass times acceleration, is 16.8 N.

F_{net} = Tension – F_{grav}

16.8 = Tension – 39.2

Tension = 56 N

Remember that we don't count that tension twice! The tension in the rope is 56 N, and it is the same no matter where along the rope we measure it.

SIMPLE HARMONIC MOTION

An object that keeps moving back and forth, like a pendulum, is a basic oscillating system. The system experiences a **restoring force** that acts against the displacement. **Harmonic motion** is a back and forth type of motion in which the restoring force is directly proportional to the displacement, which is the case for a pendulum, or a mass suspended on a spring that keeps bouncing up and down. The acceleration is not constant, because the restoring force is not constant. **Simple harmonic motion** presumes that there is no energy loss (no friction) to slow the motion down.

The **period** (T) is the elapsed time for one complete cycle of the harmonic motion. **Frequency** (f) is the number of cycles per second. 1 Hertz = 1 cycle/sec. The frequency is 1 divided by the period. That also means that you can find the period by dividing 1 by the

frequency.
Amplitude (A) is the distance from the center point, which is half of the total displacement.

When you look at an oscillating system it is actually kind of surprising to see so much motion with a single bit of input. You have to remember that energy lost due to friction is usually minimal in this situation, and what you are seeing is the result of invisible forces that change constantly depending on the object's position.

Springs

Many examples of harmonic motion involve an object oscillating on a spring, because stretching or compressing a spring creates a restoring force. We can calculate this force by using Hooke's Law:

$$F = -kx$$

By looking at this formula you can see that the restoring force is directly proportional but in opposite direction to the displacement x. This is the condition that creates simple harmonic motion. The restoring force is reduced to zero once it has corrected the displacement, but by then the moving mass has gained kinetic energy and it displaces again in the opposite direction, which creates a new restoring force.

To deform a spring requires work to be done. Since work is Force times distance, if you stretch or compress a string by a distance x, the magnitude of the work you did is the average force you used multiplied by the distance x. You want to look at average force here because the spring gets harder and harder to deform as you proceed. Fortunately the required force increases linearly, so you can just take the average between the starting force, 0, and the ending force, kx. That would be ½ kx, which is multiplied by the distance x to get $\frac{1}{2} kx^2$. This work is now stored in the spring as elastic potential energy, and it goes on to power the harmonic motion when you let go.

$$PE_{Spring} = \frac{1}{2} kx^2$$

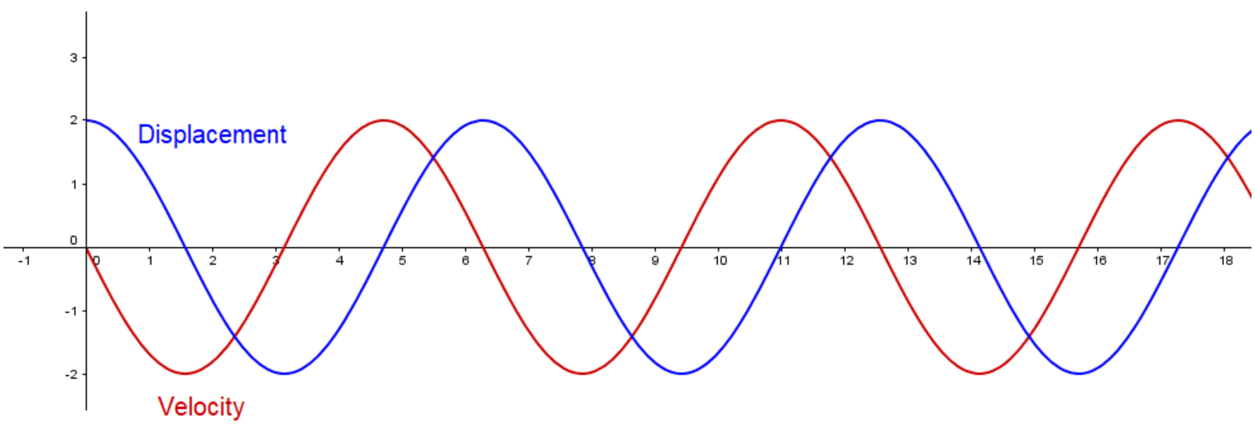
At the point in the harmonic motion where the mass attached to the spring is at the equilibrium position, **all of the energy is kinetic energy, $\frac{1}{2} mv^2$**. When the mass reaches maximum displacement, all of the energy is potential energy, and the distance x is the amplitude A of the harmonic motion: total potential energy = $\frac{1}{2} kA^2$

Example

A 56 g mass attached to spring with a spring constant of 100 N/m is oscillating horizontally on a frictionless surface, with an amplitude of 4cm. What is the maximum speed reached by the mass?

The mass reaches its maximum speed at the equilibrium position. At this point all of the spring potential energy has been converted to the kinetic energy of the mass. The maximum spring potential energy is ½ kA^2, or $(0.5)(100)(0.04)^2$. That works out to 0.08 N-m, or 0.08 Joules. This amount of energy will be converted to kinetic energy: 0.08 = ½ mv^2, so $.08 = (0.5)(.056)v^2$. The maximum velocity is 1.69 m/sec.

If you graph the displacement of a mass oscillating on a spring, you get a sine wave (or a cosine wave, depending on where you start). The velocity graph also has the form of a sine wave. The mass moves faster closer to the equilibrium position, and then slows down on either end.



The image above shows a graph for a mass on a horizontal spring that has been pulled out a distance of 2 units to the right, and then released. The initial velocity is zero. The speed increases (the velocity becomes more negative as the mass moves to the left) until the spring reaches its equilibrium position at a distance of zero. All of the potential energy has been converted to kinetic energy, so the mass has its maximum speed at this point. For an instant, no force acts on the mass and the acceleration is zero. Then the mass starts to compress the spring. The speed decreases and reaches zero at the point of maximum compression. Notice that at this point the velocity changes from negative to positive, and the mass starts to move back to the right.

The following simplified explanation will help you remember the formula for the period of a mass oscillating on a spring. A more detailed analysis using calculus is provided afterwards.

The period of a sine wave is normally 2π. For the oscillations of a spring, the period is influenced by two factors. First, if the mass is larger the period will be longer. That makes sense because the same amount of kinetic energy ($1/2mv^2$) will give a larger mass less velocity than a smaller mass. This effect is not linear, because it affects the square of the velocity rather than the velocity itself. A smaller velocity means a longer period. The second factor is the spring constant k. If the spring is very stiff k will be large. For the same amount of displacement the restoring force will be greater, since F = -kx. This larger force means more kinetic energy, which again affects the square of the velocity. There will be an increase in the velocity when k is larger, resulting in a shorter period.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Note that the period is not affected by how much you initially stretch or compress the spring. If you stretch the spring more the restoring force is greater, and it will also be acting over a longer distance, doing more work to increase the kinetic energy of the mass. The net effect on the velocity is that it is now large enough to cover the increased distance in the same amount of time, and the period stays the same.

Example

A 0.5 kg mass is oscillating on a spring with a period of 0.7 seconds. Another mass of 0.5 kg is released from some distance above the equilibrium point of the oscillating mass-spring system. It collides with the first mass and sticks to it. a) Calculate the new period. b) The amplitude of the oscillation changes after the collision. How does it compare to the previous amplitude?

In this example, there is an inelastic collision. Momentum is conserved, but there is a loss of kinetic energy. Because we are making a change at a critical point where all of the energy is in one form (kinetic energy), it is like starting a new harmonic motion with new conditions.

a) The period is affected only by the mass, as shown in the equation $T = 2\pi\sqrt{\frac{m}{k}}$. You can find the spring constant k, and then calculate the new period, but it is much easier to simply substitute 2m for m:

$$T_1 = 2\pi\sqrt{\frac{m}{k}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{2m}{k}}$$

$$T_2 = 2\pi\sqrt{2 \cdot \frac{m}{k}}$$

$T_2 = 2\pi\sqrt{2}\sqrt{\frac{m}{k}}$

$T_2 = \sqrt{2}\, T_1$

$T_2 = \sqrt{2} \cdot 0.7 = 0.99$ sec

b) The amplitude is determined by how much energy is available to compress the spring. That energy is supplied by the kinetic energy of the mass at the equilibrium point, which is converted to spring potential energy at maximum amplitude. Because momentum is conserved, we can say that $0.5\, v_1 = 1.0\, v_2$. The new velocity is half of the previous velocity. The initial kinetic energy is ½ (0.5) $(v_1)^2$, and the new kinetic energy is ½ (1.0) $\left(\frac{v_1}{2}\right)^2$. This means that the initial kinetic energy is $\frac{1}{4} v_1^2$ and the final kinetic energy is $\frac{1}{8} v_1^2$. As you might expect, the kinetic energy has been cut in half. The spring potential energy at maximum amplitude is ½ kA^2, so that will also be cut in half. The spring constant k can't change, so A_2^2 must have half the value of A_1^2:

$A_2^2 = \frac{1}{2} A_1^2$

$A_2 = \sqrt{\frac{1}{2} A_1^2}$

$A_2 = \sqrt{\frac{1}{2}}\, A_1 = 0.707\, A_1$

<u>Calculus</u>

To find the period for a mass oscillating on a spring, we can start with the fact that the position vs time graph must be a sine or cosine wave, as can be shown experimentally. To model the motion, we may want to start at the point where we pull the spring out and then let go. In this case the maximum amplitude occurs at t = 0, so we will use a cosine function. It will be of the form x = cos(t), where x is the position. To get the correct amplitude we must multiply by A: x = A cos(t). The period of that is 2π, but we want it to be T. To get the period to be 1, we multiply t by 2π, and then divide t by T so that our function has a period of T: $x = A \cos\left(\frac{2\pi}{T} t\right)$*. This is the position function.*

Next, we want to find an expression for T that incorporates the two things known to influence the period: m and k. Force equals mass times acceleration, and we also know the force that drives the motion. It is the spring force, $F_{Spring} = -kx$*. So,* $m a = -k x$*. Because x and a are* *constantly changing, and not in a nice linear way, we need calculus. We can calculate the acceleration a because it is the derivative of the velocity function, and the velocity in turn is the derivative of the position function. To get a good expression for a, we need the second derivative of* $x = A\cos\left(\frac{2\pi}{T}t\right)$*. Use the Chain Rule:*

Position: $x = A\cos\left(\frac{2\pi}{T}\,t\right)$

Velocity: $x' = A \cdot -\sin\left(\frac{2\pi}{T}\,t\right) \cdot \frac{2\pi}{T}$

Acceleration: $x'' = \frac{2\pi}{T} A \cdot -\cos\left(\frac{2\pi}{T}\,t\right) \cdot \frac{2\pi}{T}$

Now use this expression for the acceleration a:

$m\,a = -k\,x$

$m \cdot -\left(\frac{2\pi}{T}\right)^2 A\cos\left(\frac{2\pi}{T}\,t\right) = -k\,A\cos\left(\frac{2\pi}{T}\,t\right)$

$m \cdot \left(\frac{2\pi}{T}\right)^2 = k$

$\left(\frac{2\pi}{T}\right)^2 = \frac{k}{m}$

$\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

$\frac{T}{2\pi} = \sqrt{\frac{m}{k}}$ so $T = 2\pi\sqrt{\frac{m}{k}}$

Vertical Springs

Most spring problems are presented as a horizontal motion on a frictionless surface. It is also possible to look at vertical springs in harmonic motion. **The net influence of gravity is a change in the equilibrium position.** When a mass simply hangs on a vertical spring, the force of gravity on the mass is balanced by the spring's restoring force:

$kx = mg$

This allows us to calculate the distance x at which the new equilibrium position occurs. If the spring is pulled down further, by a distance A, and then released, the spring will pull the mass up and gravity will pull the mass down. There will be a net upward force on the mass with a magnitude of kA. We can see that from the sum of the forces. Taking up as positive and down as negative, we get:

$F_{spring} + F_{grav} = k(x + A) + -mg$

$F_{spring} + F_{grav} = kx + kA - mg$ and since $kx - mg = 0$:

$F_{net} = kA$

Now let's see what happens if the spring is instead compressed by a distance A and released. If A is smaller than x, the spring is still in a stretched state and will pull upward, but the force of gravity will be larger than the spring force. If A is larger than x the spring force is directed downward.

$F_{spring} + F_{grav} = k(x - A) + -mg$

$F_{spring} + F_{grav} = kx - kA - mg$

$F_{net} = -kA$

The net force is downward, with a magnitude of kA.

The restoring force is still directly proportional to the displacement. At any distance a the magnitude of the spring force is ka. With either an initial upward or downward displacement of the mass from the equilibrium position by a distance A we get simple harmonic motion with an amplitude of A, just as for a horizontal spring. The amplitude is the same, but the equilibrium position is different.

Example

A 56 g mass attached to spring with a spring constant of 100 N/m is oscillating vertically with an amplitude of 4.00 cm. What is the maximum speed reached by the mass?

This problem is really the same as the first example we looked at for the horizontal spring. We know that at the maximum amplitude A, all of the energy is spring potential energy. The restoring force at this point has a magnitude of kA, and it decreases linearly to 0 at the equilibrium position. The work done by the spring over this distance is $\frac{kA-0}{2} \cdot A = ½\, kA^2$. All of that goes into increasing the kinetic energy: $½\, kA^2 = ½\, mv^2$. $(0.5)(100)(.04)^2 = (0.5)(.056)v^2$. That works out to a maximum velocity of 1.69 m/sec, just as for the problem in the last section.

Hmmm, is that really true? Here is an in-depth look at what is happening when we consider the influence of gravity:

At the equilibrium position, $kx = mg$. This is the point where the mass will have maximum kinetic energy, and therefore maximum velocity. In this case $100\,x = (.056)(9.8)$, so $x = .00549$ m or 0.549 cm. The lowest point of the oscillation will be at 4.549 cm. At this point the gravitational potential energy can be considered to be 0, while the spring potential energy is $½k(x + A)^2$ or $(.5)(100)(.04549)^2 = 0.1035$ J. At the equilibrium position, 4 cm higher, the spring will still have spring potential energy, $½\,kx^2$, or $(.5)(100)(.00549)^2 = 0.001507$ J. The spring has lost 0.1035 – 0.001507 or 0.10199 J. However, the system has gained mg(A) or (.056)(9.8)(.04) = .02195 J. of gravitational potential energy. Only 0.10199 – 0.02195 or 0.080 J is available to increase the kinetic energy of the mass. $½\,(0.056)v^2 = 0.08$. The maximum velocity is 1.69 m/sec, just like for the horizontal spring.

Notice that what we did algebraically is $½\,k(x + A)^2 – ½\,kx^2 – mgA$ = Available Potential Energy.

$½\,k(x^2 + 2Ax + A^2) – ½\,kx^2 – mgA = ½\,mv^2$

$½\,kx^2 + kAx + ½\,kA^2 – ½\,kx^2 – mgA = ½\,mv^2$

$kAx + ½\,kA^2 – mgA = ½\,mv^2$ And since $kx = mg$,

$kAx + ½\,kA^2 – kxA = ½\,mv^2$

$½\,kA^2 = ½\,mv^2$

The additional calculations here show that things work out the same for vertical springs when you just need the maximum velocity.

Practice: Start from the highest point of the oscillation and show that the maximum velocity is 1.69 m/sec.

Example

A 200 g mass is dropped on top of a vertical spring that has a spring constant of 150 N/m. The velocity of the mass just before it hits the spring is 3.5 m/sec. How far does the spring compress? How far above its resting length will it bounce back?

In this example, the kinetic energy and potential energy of the mass will be converted to spring potential energy. The kinetic energy of the mass is $½\,mv^2$, and since it stops momentarily at the point of maximum compression, its kinetic energy there will be zero. Gravity will also contribute. The potential gravitational energy of the mass is mgh, and at the point of maximum compression we will say that h is zero. h will be the distance the spring compresses.

$PE_{Spring} = ½\,mv^2 + mgh$

PE_{Spring} is $½\,kx^2$, which is $½\,kh^2$ here

$½\,kh^2 = ½\,mv^2 + mgh$

$\frac{1}{2}\ 150h^2 = \frac{1}{2}\ (0.2)(3.5)^2 + (0.2)(9.8)h$

$75h^2 = 1.225 + 1.96h$

$75h^2 – 1.96h – 1.225 = 0$

The quadratic formula yields a positive answer for h of .1415m, or 14.1 cm. This is how much the spring compresses.

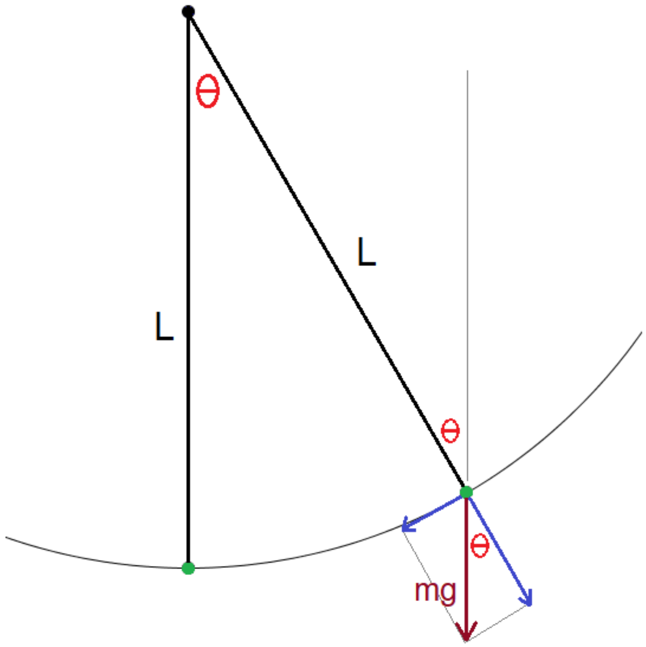
The equilibrium position in this case is mg = kx, or (0.2)(9.8) = 150 x. This gives a distance x of 0.0131 m, or 1.3 cm of compression. The spring will compress 0.1415 – 0.0131 = .1284 cm below the equilibrium position, so it will bounce back with the same amplitude of 12.8 cm above equilibrium.

Note that the negative solution to the above quadratic equation, -11.5 cm, represents how far the spring will bounce upward above the resting position. You can find the equilibrium position directly from the two solutions as the average displacement: (14.1 + -11.5) ÷ 2 = 1.3 cm.

Practice: Start with the fact that the spring compresses .142 m, and calculate the impact velocity of the 200 g mass.

Motion of a Pendulum

For a pendulum the restoring force is provided by gravity. In the image below, you can see that the force pulling the pendulum back to equilibrium is mg sin θ. (Notice that we can mark several angles as θ because they are either alternate interior angles or vertical angles.)



The gravitational force mg has been decomposed into a restoring force, and a force that adds to the tension in the string. (The tension in the string will be greater than that depending on the velocity, because the mass is undergoing circular motion – see Tension).

The actual displacement of the pendulum, which we will call x even though it lies along an arc, is the arclength Lθ, where L is the length of the string (the radius of the circle). To get simple harmonic motion, the restoring force has to be directly proportional to the displacement. Unfortunately, here the restoring force is mg sin θ, while the displacement is Lθ. The angle is the only changing quantity, but sin θ is not directly proportional to θ as would be required. We can get around this by specifying that the angle must be "small", in which case sin θ is nearly equal to θ. That works quite well up to about 15^{o} (0.2618 radians), where the sine is 0.2588. The angle shown in the image is actually about 30^{o}.

Due to this not being the best example of simple harmonic motion, we looked at springs first. We will use the same model – a perfect sine wave – to analyze pendulum motion, and make it fit by using a small displacement.

For the spring, the restoring force was perfectly proportional to the displacement: F = -kx. Here k is the spring constant, but really any constant will do. For the pendulum at small displacements, the restoring force is F = -mg θ (using θ instead of sin θ), while the displacement x is Lθ. We can say that $\theta = \frac{x}{L}$, so we can write the restoring force as $F = -mg\,\frac{x}{L}$, or preferably as $F = -\frac{mg}{L}\,x$. Now you can see that the restoring force is directly proportional to the displacement x. We have simple harmonic motion!

Previously we used a sine wave to model simple harmonic motion, and found that the period T was T $= 2\pi\sqrt{\frac{m}{k}}$, using the parameters for the spring. The constant of proportionality is for the pendulum is $\frac{mg}{L}$ rather than k, so we plug that into our previous formula:

$$T = 2\pi\sqrt{\frac{m}{mg/L}}$$

m cancels out, and division by g/L is the same as a multiplication by L/g:

$$\mathbf{T = 2\pi\sqrt{\frac{L}{g}}}$$

Because the force of gravity on a big mass is larger than the force of gravity on a small mass, objects of different weights fall at the same rate (not counting air resistance). Remember that things don't just fall magically. When a mass is large, gravity has to do more work to pull that mass down. Mass has no effect on the period of a pendulum!

The actual period of a pendulum on Earth is determined by the length of the string. That makes sense, because for the same horizontal distance a longer string produces less of a vertical displacement. Gravity then does less work pulling the mass down, which means the mass gains less kinetic energy. This affects the square of the velocity, so the pendulum moves more slowly. T increases. If you were able to move the pendulum to a more massive planet, g would be larger and your pendulum would move faster, decreasing T. These considerations will help you remember the formula.

Waves

A wave is a disturbance that carries energy from one location to another. The **medium** is the substance through which a wave travels. A **pulse** is a single disturbance that travels through a medium. A series of pulses form a wave. A **periodic wave** is a disturbance that is produced continuously in a regular pattern. The time it takes for a wave to complete one cycle is the **period**, T. You can measure the length of the period anywhere, such as from peak to peak. The period is the inverse of the **frequency**, which is the number of cycles per second. The frequency of a wave depends on the source, which produces the oscillations.

$$\boldsymbol{f = \frac{1}{T}}$$

The speed of the wave can be found by multiplying the number of cycles per second by the wavelength λ:

$$\mathbf{v} = \boldsymbol{\lambda f}$$

Transverse waves cause the particles of the medium to move perpendicular to the direction of the wave's motion. The particles of the medium are not carried along by the propagating wave.

Longitudinal waves: particles go back and forth in the same direction as the wave. They are not carried along by it. There are more dense areas called compressions, and less dense areas called rarefactions.

Standing waves occur when a wave is reflected back toward the source, or whenever two waves moving in opposite directions have the exact same frequency. There are **nodes** where the wave is cancelled by its reflection. The nodes don't move. The **antinodes** are the points that show maximum displacement.

The speed of a wave does not depend on the frequency or wavelength. In a given situation, when the frequency increases the wavelength just decreases and vice versa, without changing the speed. Light and other electromagnetic waves move at the speed of light, approximately 300,000 kilometers per second, in a vacuum. A transverse wave moves more slowly in a denser medium (greater mass). Light waves move at about 225,00 km/sec in water. However, sound waves are longitudinal waves that move faster in a denser medium like a liquid or solid because it has more "stiffness". The compressions and rarefactions are transmitted faster. Sound, which has a speed of about 343 m/sec in air, can travel at up to 12,000 m/sec through an exceptionally stiff medium such as diamond.

We can determine the speed of a wave traveling along a string (a transverse wave) if we know the tension in the string and its **linear mass density**, μ. The linear mass density of a string is found by dividing the weight of the string by its length. The speed of the wave is given by

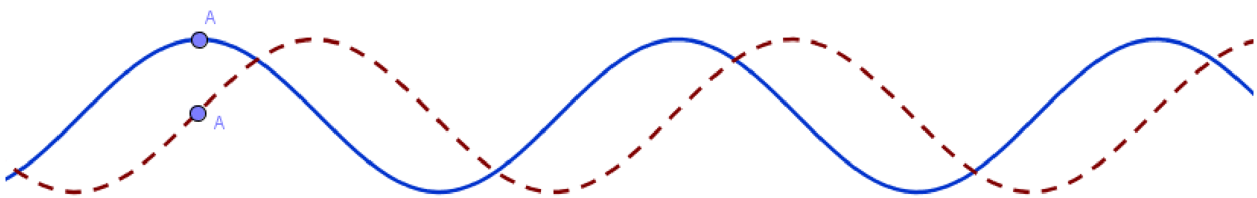
$$v = \sqrt{\frac{F_T}{\mu}}$$

where F_T is the tension in Newtons.

Displacement of Points on a Moving Wave

Because the particles of the medium don't move longitudinally along with a transverse wave, it can be bit confusing to analyze their motion. In order for the wave to propagate, all of the particles have to undergo an up and down motion.

Consider a point A marked on a rope. As a wave travels along the rope, point A will be lifted up until it reaches its maximum displacement. This is the **amplitude** of the wave, which is the maximum distance from the resting (equilibrium) position. In the diagram, point A is at the crest of the wave at this point in time.



As the wave moves along the rope, point A accelerates back down toward the equilibrium position. At this time it has its maximum velocity, and its acceleration is zero for an instant. It overshoots the equilibrium position and begins to slow down, which means the acceleration vector points in the direction opposite its motion. Eventually it reaches zero velocity at the trough of the wave. Then it reverses direction and gains speed (acceleration is in the same direction as the motion) as it approaches the equilibrium position again.

To determine the direction in which the wave is moving, you need to look at the relative motion of at least two points. Wave diagrams typically show only a single point in time. Don't hesitate to sketch in the position of the wave a few moments later so you can see which way things are going.

Sound

Sound waves are longitudinal pressure waves, although they can have a transverse component in solids. Sound cannot be transmitted in a vacuum. Speakers produce sound through the vibrations of a diaphragm. Larger excursions of the diaphragm produce louder sounds, while more frequent vibrations produce a higher **pitch**. Pitch is the perceived highness or lowness of a particular sound.

Sound intensity is measured in **decibels (dB)**. The decibel scale is a type of logarithmic scale that is set up to reflect the way that our ears respond to sound. Zero decibels corresponds to the smallest intensity of sound that humans can hear. Ten decibels is 10 times more intense than that, 20 decibels is 100 times more intense, and 30 decibels is a sound that has 1000 times the power. The intensity of the sound goes up a lot for each 10 decibels of increase, but it doesn't seem that much louder to us. A whisper is about 20 - 30 dB, while normal conversations are at about 60 dB. Exposure to sounds around 140 dB causes immediate pain and hearing damage, but much less intense sounds can also damage your hearing if you are exposed to them for longer periods.

Sound travels the slowest through a gas, faster in a liquid, and fastest through a solid. The speed of sound in room temperature air is about 346m/sec. Sound travels faster at higher temperatures. When a sound wave hits a different medium, there is a partial reflection. The wave continues in the new medium at the same frequency, since the frequency depends on the source. However, the velocity and therefore the wavelength will be changed, and the energy of the wave has been reduced due to the reflection. The sound has lost some of its intensity, so it will be less loud.

Even though sound waves are longitudinal waves, and they spread out in all directions from the source, they are usually represented as simple sine waves in diagrams. In these illustrations the maximum positive part of the sine wave represents a **compression** (maximum density), while the maximum negative part represents a **rarefaction** (area of minimum density).

Harmonics are integer multiples of a particular basic frequency, which is called a **fundamental** or the first harmonic. The second harmonic has a frequency that is double the frequency of the fundamental. The third harmonic has triple the frequency, and so on.

Standing waves are formed on the strings of musical instruments, since these strings are fixed at both ends and the waves reflect. A standing wave is really two waves with the same wavelength moving in opposite directions and coinciding perfectly. The longest standing wave that fits on a guitar string has a wavelength that is twice as long as the length of the string (the wave reflects after half its period). This is the lowest possible frequency – the fundamental. There are nodes (no movement) at both ends. and the string vibrates up and down between them. The center of the string is an antinode (maximum movement). The second harmonic has twice the frequency, so it is a shorter wave with a wavelength equal to the length of the string. It has three nodes: two at each end and one in the middle. The third harmonic has four nodes, and 1.5 of its wavelengths fit on the string. By the time we reach the fourth harmonic two full wavelengths form a standing wave on the string.

Standing sound waves are also formed inside tubes or pipes, even if they are open at both ends! However, instead of nodes we find points of maximum movement at the open end(s). If there is a closed end it is a node. A standing wave inside a tube that is closed on one end can have a wavelength that is four times as long as the length of the tube (it reflects off an end four times before it completes one wavelength). The distance from a node to the first point of maximum displacement is really only a quarter of the full period of a wave. The next higher frequency wave that will fit in such a tube must have a node at the closed end, then another node, and then a point of maximum displacement at the open end. That takes up ¾ of a full wave period, and so it is called the third harmonic. There are no even-numbered harmonics in this case, and the next harmonic has $\frac{5}{4}$ of a wave inside the tube. Although we are not talking about whole waves, there are actually 5 times as many waves in the tube at this point than there were for the first harmonic. The number of waves in the tube (the frequency) for the first three harmonics are in the ratio 1 : 3 : 5. Correspondingly, the wavelengths are in the ratio $1 : \frac{1}{3} : \frac{1}{5}$. The frequencies are easier to work with for problems involving harmonics. Divide the given frequencies by the smallest number, and then multiply the ratio by consecutive integers until you get whole numbers.

The **Doppler Effect** occurs when there is relative movement between the source of a sound and its detector. If an ambulance or fire truck is moving towards you, it travels a small distance forward between each pulse of the sound wave. This results in the waves being closer together, so the frequency is higher (and the wavelength is shorter). Once the emergency vehicle passes, the waves are further apart and you hear a lower frequency. On the other hand, if a sound source is stationary and you are moving toward it the wavelength is unchanged but you perceive a higher frequency because you encounter the waves more frequently as you move. It does make a difference whether the observer or the source is moving, as you can find by playing around with the formula for the Doppler Effect:

Observed frequency: $\mathbf{f_0 = f\frac{v \pm v_0}{v \pm v_s}}$

Here v_o is the velocity of the observer, and v_s is the velocity of the source. You have to adjust the signs based on whether the distance is increasing or decreasing (consider whether you expect the observed frequency to be higher or lower).

All of this is assuming constant speeds. For multiple choice questions, pay attention to whether the source is slowing down or speeding up. If an ambulance moves toward you, you hear a higher frequency, but that frequency will decrease if the driver hits the brake.

ELECTRICITY

Electric charge is carried by protons and electrons. Since electrons are more mobile, your problems will usually involve these negative charges. Electrons may redistribute unevenly, so that a static charge builds up.

Positive and negative electrical charges attract each other, and that attraction gets stronger quickly as they get closer together, just like gravitational attraction. Unlike gravity however, electrostatic force can be repulsive, with like charges experiencing a force that pushes them apart. Electrostatic force is also much stronger than gravitational force. Coulomb's Law gives the magnitude of the force:

$$F = k\frac{q_1 q_2}{r^2}$$

Here F is in Newtons, and q is the electrical charge, measured in **Coulombs** (C). 1 Coulomb of electrical charge is equivalent to about 6.24×10^{18} electrons. k is Coulomb's constant, about 8.988×10^9 N-m^2/ C^2. Just like the force of gravity, electrostatic force decreases with the square of the radius. When the two charges have opposite signs they attract, and F will have a negative value.

Just like a mass creates a gravitational field, a charge Q creates an electrical field around itself. The magnitude of this electrical field is $k\frac{Q}{r^2}$. Electrical field strength is measured in Newtons/Coulomb. By convention, representative arrows in an electrical field point from positive to negative. A higher density of field lines indicates greater field strength.

Electrons can move continuously, creating an electric current. A current of 1 **amp** means that 1 Coulomb of charge (6.24×10^{18} electrons) is passing by a given point every second. A mole of electrons is 6.022×10^{23} electrons, which is equivalent to 96,485 Coulombs of charge. Due to an unfortunate convention, current is considered to flow from positive to negative, which is opposite to the actual flow of electrons.

In the case of **direct current (DC)**, electrons move in one direction only. With **alternating current (AC)**, the electron flow reverses many times per second. The power in your home is AC current, reversing at a frequency of 60 cycles per second (60 Hz) if you live in the US. The advantage of alternating current is that it can be transported over longer distances at high voltage, and then stepped down to a lower voltage through the use of **transformers**. Transformers can also convert high voltage alternating current to the low voltage direct current required by many electronic devices.

To create a current, we need an electrical potential, the strength of which is expressed in **volts**. One volt is the electrical potential difference that imparts 1 Joule of energy to a charge of 1 coulomb. The voltage is a measure of the "pull" on the electrons, much as the height of a waterfall affects the water that goes over it.

Metals conduct electrical currents readily, but even metal wires provide some resistance (**R**). The greater the resistance is, the less current flows. There is a noticeable difference between wires composed of different metals. Thinner wires provide more resistance than thicker wires, since a wire with a large diameter can conduct more current. Of course the longer the wire is the more resistance there will be, although when I tried I found that you have to increase the length of a metal wire quite a lot to get a measurable difference. All of this is important in practical applications, so we have a formula which is relatively simple in the case of direct current:

$$\mathbf{R = \frac{\rho L}{A}}$$

where L is the length of the wire and A is the cross-sectional area. ρ is the **resistivity** of the material. The **conductivity** of a particular material is $\frac{1}{\rho}$. Electrical resistance is higher for AC current.

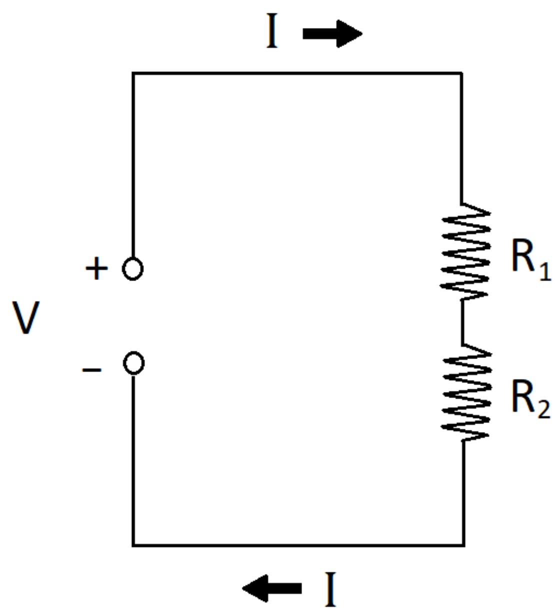
To control current flow, resistors are often used in electrical circuits. You can recognize these small cylindrical components by the colored bands on them that code the amount of resistance. Resistance is measured in **Ohms (Ω)**. Ohm's Law says that the flow of current (**I**) increases with the voltage and decreases with increasing resistance:

$$\mathbf{I = \frac{V}{R}}$$

That makes a lot of sense, and this is the best way to write Ohm's Law so that you'll remember it.

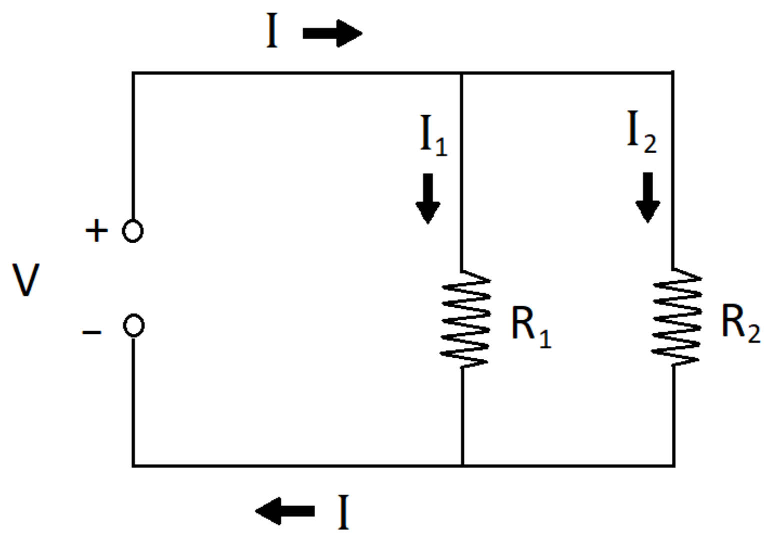
Light bulbs also add resistance to a circuit, and using small low-voltage light bulbs is a good way to see the effects of that. (I use bulbs cut from strings of battery-operated lights.) If you connect a single light bulb to a battery it shines brightly, but when you connect two bulbs **in series**, they are both dimmer. The increase in resistance causes less current to flow through the circuit.

Two resistors connected in series are shown below. $R = R_1 + R_2$:



Because there is only a single loop, the current is the same everywhere in the circuit. The voltage however is not. There is a **voltage drop** across each resistor. You can use Ohm's law to calculate the magnitude of this drop. Since $I = \frac{V}{R}$, V = IR. For example, if the battery in this circuit supplies 9V, and the resistors are 200 Ohms and 400 Ohms, the current is $\frac{V}{R}$, or $\frac{9}{600}$ = 0.015 Amps. That current flow is the same in both resistors. For the 200 Ω resistor, the voltage drop is 0.015 x 200 or 3 Volts. For the 400 Ω resistor the voltage drop is 0.015 x 400 = 6 V. The wires are considered to have negligible resistance.

Resistors may also be connected **in parallel**. When you do that, you create an extra path, so more current can flow. The total resistance decreases. If you connect two small lightbulbs to a battery in parallel, they will be much brighter than if they were connected in series. Two resistors connected in parallel are shown below.



The flow of current splits, but the individual currents have to add up to the total. $I = I_1 + I_2$. This common-sense idea is called **Kirchhoff's Junction Rule**. The total current that flows into a junction has to equal the total current flowing out of a junction.

Ohm's Law tells us that $I = \frac{V}{R}$, and since $I = I_1 + I_2$, we can say that $\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$. Note that the voltage V is the same across both resistors. Now we can divide both sides of the equation by V:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

This is the easiest way to remember the formula for total resistance in parallel circuits. If there are more resistors connected in parallel, just extend it:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots.$$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ can be rearranged like this:

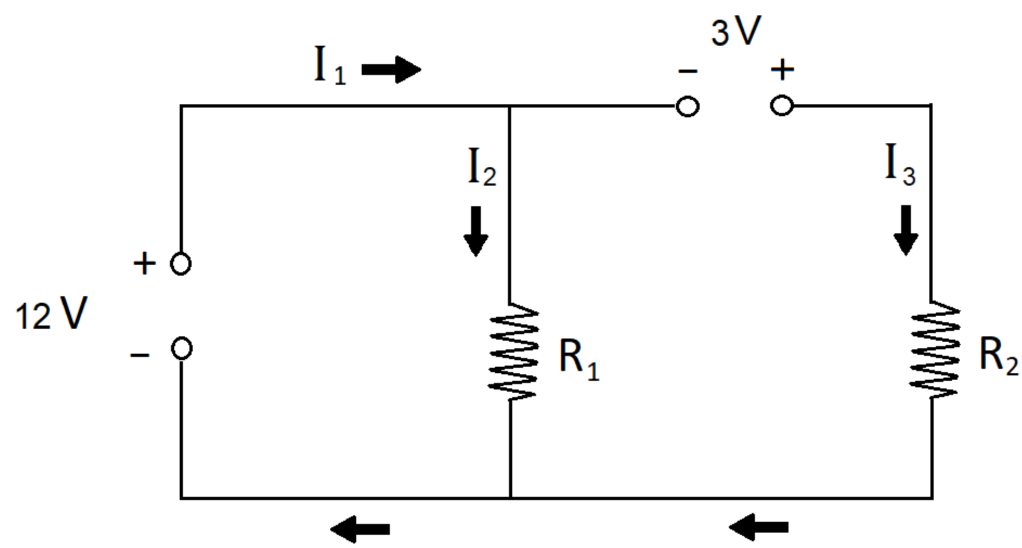
$$\frac{1}{R} = \frac{R_2}{R_1R_2} + \frac{R_1}{R_2R_1}$$

$$\frac{1}{R} = \frac{R_1 + R_2}{R_1R_2}$$

$$R = \frac{R_1R_2}{R_1 + R_2}$$

Current flow through parallel resistances is a matter of proportion. If the resistances are not equal, more current will flow through the branch with the lowest resistance. If you connect a 4000 Ω and a 5,000 Ω resistor in parallel, the current flows in a ratio of 5 : 4 in the two paths.

For some problems you may be asked to determine current flow in a bizarre circuit that contains two voltage sources and a few resistors.



This circuit has no practical use other than to let you do calculations involving Kirchhoff's Junction Rule and voltage drops. To solve problems of this type, first pick a current direction. I always look at the highest voltage source, and ignore the lower one, which is often connected in the opposite direction. The arrows in the picture show the current flow being driven by the 12 V source in a logical way. If you do end up picking the wrong direction it will work out negative in the end so it is not a problem. Use the junction rule to say that $I_1 = I_2 + I_3$. That gives you three unknowns, but we can just say that $I_3 = I_1 - I_2$. Next, consider that the voltage in each separate loop of the circuit is "used up" by the voltage drops. Consider the original voltage as positive, and the voltage drops as negative. Where the current direction crosses the smaller voltage source, list the voltage as negative if you go the wrong way across (from negative to positive), as there will be a voltage drop. Otherwise, list the voltage as positive because it will give a boost to the flow of current.

$I = \frac{V}{R}$, so the voltage drop across a resistor is given by IR. List it as negative, unless you are going around a loop and see that you are moving against the current. First we will look at the loop with the 12 V source and R_1. The net voltage is zero around the loop: $12V - I_2R_1 = 0$. If you know R_1, you can calculate I_2. However, problems may have more resistors and force you to create a system of equations. Next, let's consider the large outer loop. Again the net voltage must be zero: $12V – 3V – I_3R_2 = 0$. Note that we are subtracting 3V because our listed current flow is going across the 3V source from negative to positive. For the smaller loop on the right, the equation is $-3V – I_3R_2 + I_2R_1 = 0$. As you move around this loop, you are going against the direction of current flow when you cross R_1, so list the voltage drop as positive. For this circuit, if $R_1 = 100\Omega$ and $R_2 = 150\Omega$, I_2 will be 0.12A, I_3 will be 0.06A, and I_1 is the sum, 0.18A.

As you may or may not recall, Power is the work done per unit of time. It is measured in Joules per second. Since one volt imparts 1 Joule of energy to 1 coulomb, and current is measured in coulombs per second (amps), we can say that if the current is 1 amp and the voltage is 1 Volt, the Power is 1 Joule/sec. Electrical power is measured in Watts: 1 Watt = 1 Joule/sec. That goes up if we increase the voltage, and also if we increase the current: electrical power = voltage x current.

$P = I V$

Since V = IR, we can also say that Power = I^2R. Power is what determines the brightness of a light bulb. A 60 Watt bulb uses 60 Joules of power every second. At a standard voltage of 120V, the current flowing through that bulb is 0.5 Amps. Since your power company is interested in how much power you use in total, they keep track of kilowatt hours (1 kWh = 1000 Watts per second for one hour).

MAGNETISM/ELECTROMAGNETISM

People first became familiar with magnetism through lodestones, which are naturally occurring pieces of the mineral magnetite, an iron oxide compound. It is thought that lightning strikes are responsible for turning this mineral into permanent magnets. Lodestones were used to magnetize compass needles.

All magnets have a north pole and a south pole, which cannot exist separately. One magnet's north pole attracts another magnet's south pole. The north pole of a magnet is attracted to the Magnetic North Pole of the Earth. Earth is a large (but fairly weak) magnet, with its south pole located at the Magnetic North Pole.

Ferromagnetic metals such as iron, nickel and cobalt are most strongly attracted to magnets. **Paramagnetic** materials are very weakly attracted to magnets, and **diamagnetic** materials are repelled by magnets (usually very weakly). Permanent magnets are created from hard magnetic materials, which are more difficult to magnetize but retain their magnetism well. Soft magnetic materials form temporary magnets.

Magnetic fields are produced by moving electrical charges. We can see this when we bring a compass near a wire carrying current. We can also create electromagnets, which function while current is flowing. At the atomic level, the quantum spin of electrons (and to a lesser degree their movement around the nucleus) is responsible for the magnetic effects seen in various substances. When an atomic orbital is full, there are an equal number of electrons with up spins and down spins. Magnetic materials have unpaired electrons, and their unpaired spins can line up to produce very small magnetic domains within the material. An external magnetic field can cause these tiny domains to line up, which turns the material into a magnet.

Magnets create a magnetic field, which is represented in textbooks by magnetic field lines. Iron filings spread near a magnet show these field lines very dramatically, but the "lines" are actually due to clumps of magnetized filings repelling each other. The magnetic field does not consist of lines, and it is 3-dimensional.

LIGHT/OPTICS

Visible light is a small part of the **electromagnetic** spectrum. Electromagnetic waves travel at the speed of light (c), which is approximately 300,000 kilometers per second. Their frequency and wavelength can be calculated from the equation $\mathbf{c = \lambda\nu}$ where λ (the Greek letter l) is the wavelength, and ν (the Greek letter n) is the number of waves per second (the frequency). A frequency of 1 Hertz means 1 wave per second.

Radio waves have a very long wavelength, and x-rays have a very short wavelength. When the wavelength is short the frequency is high, and the radiation is high in energy according to the equation $\mathbf{E = h\nu}$. Here E is the energy and h is Planck's constant, 6.626 x 10^{-34} Joule sec. (1 Joule = 1 kg·m^2/sec^2). You can combine the two equations $c = \lambda\nu$ and $E = h\nu$ to get $\mathbf{E = h \cdot \frac{c}{\lambda}}$.

White light is a combination of photons of different wavelengths, and if we pass it through a prism we see these different wavelengths as separate colors. We are able to detect these colors because our eyes have cones that respond to different wavelengths. Our cones detect red (longest wavelength), green (intermediate wavelength) and blue (shorter wavelength) light. If the wavelength is longer or shorter than that we don't see anything, so we cannot see x-rays or radio waves. Different combinations of the **primary colors** (red, green, and blue) create the other colors that we can see.

To display color on a screen we use red, green and blue subpixels to make up a pixel. The actual color of a pixel depends on the intensity of light displayed by each subpixel. There are 256 different intensities that each red, green or blue subpixel can display. That means values range from 0 to 255. You should use a search engine to find an RGB color picker so you can experiment. 0, 0, 0 will be black (no light at all), while 255, 255, 255 will be white (maximum light from all colors). Red light and green light add to make yellow light: RGB 255, 255, 0 is yellow. Note that this is an **additive color** scheme, with each value adding up to create a specific color. You should verify that green and blue add to make cyan, while red and blue add to magenta. Yellow, cyan and magenta are brighter colors than red, green and blue because there is more light reaching your eyes.

Dyes, paints and printer ink use pigments that appear to be a particular color because they absorb light of other colors. Green paint looks green because when light strikes the pigments all wavelengths except green are absorbed. Only the green light is reflected back, so you see the color green when you look at the paint. To create colors, your printer uses magenta, cyan and yellow ink. These are the brightest colors, so they can be used as primary colors. When you add two colors of ink together, more wavelengths will be absorbed and the resulting color is darker. This is a **subtractive color** scheme. Using RGB notation, yellow *ink* could be represented as 0, 0, -255. When you add white light (255, 255, 255), the blue wavelengths are neutralized. You get 255, 255, 0, which is yellow. Cyan ink looks pale greenish blue to you because it absorbs red light, leaving blue and green to reflect back at you. Mixing yellow and cyan ink creates green: 0, 0, -255 + -255, 0, 0 = -255, 0, -255. Add white light to get 0, 255, 0. If you mix yellow, cyan and magenta together, all colors are absorbed so you will see black.

FLUIDS

Fluids are substances that can flow and change their shape. Both liquids and gases are considered fluids.

The density of a substance is found by dividing its mass by the volume it is occupying:

$$\rho = \frac{\text{mass}}{\text{volume}}$$

The units of density are kg/m^3

Fluids exert an upward **buoyant force** on objects that may cause them to float. According to **Archimedes' Principle**, the magnitude of this buoyant force is equal to the weight of the fluid that is displaced by the object.

Buoyant Force = mass of displaced fluid · g

$$= \text{volume of displaced fluid} \cdot \frac{\text{mass of displaced fluid}}{\text{volume of displaced fluid}} \cdot g$$

Since density is $\frac{\text{mass}}{\text{volume}}$,

Buoyant Force = volume of displaced fluid · density of fluid · g

The buoyant force opposes the force of gravity on the object. If something is floating, it is in equilibrium with respect to these two forces. In that case the force of gravity must be equal to the buoyant force. Even if the object doesn't float, it will appear to weigh less:

$$\mathbf{F_{net} = F_{buoyant} - F_{gravity}}$$

$$= \text{mass}_{\text{displaced fluid}} \cdot g - \text{mass}_{\text{object}} \cdot g$$

We can consider the density of a submerged object and the density of the fluid like this:

$$= \text{volume}_{\text{displaced fluid}} \cdot \frac{\text{mass}_{\text{displaced fluid}}}{\text{volume}_{\text{displaced fluid}}} \cdot g - \text{volume}_{\text{object}} \cdot \frac{\text{mass}_{\text{object}}}{\text{volume}_{\text{object}}} \cdot g$$

$$= \text{volume}_{\text{displaced fluid}} \cdot \text{density}_{\text{fluid}} \cdot g - \text{volume}_{\text{object}} \cdot \text{density}_{\text{object}} \cdot g$$

The volume of the object and the volume of the displaced fluid are equal for a submerged object, so let's call it V:

$$F_{net} = V \cdot \text{density}_{\text{fluid}} \cdot g - V \cdot \text{density}_{\text{object}} \cdot g$$

$$F_{net} = (\text{density}_{\text{fluid}} - \text{density}_{\text{object}}) \cdot V g$$

If the object is denser than the fluid it will sink. The net force will be negative (downward).

There is a simple relationship between the buoyant force and the gravitational force, which you may need to solve problems:

$F_{net} = F_{buoyant} - F_{gravity} = V \cdot density_{fluid} \cdot g - V \cdot density_{object} \cdot g$

$F_{buoyant} - F_{gravity} = V \cdot density_{fluid} \cdot g - V \cdot density_{object} \cdot g$

$$\frac{F_{gravity}}{F_{buoyant}} = \frac{Vg\ density_{object}}{Vg\ density_{fluid}}$$

$$\mathbf{\frac{F_g}{F_b} = \frac{density_{object}}{density_{fluid}}}$$

Pascal's Principle says that if you put pressure on an enclosed fluid, that pressure will be transmitted unchanged through the fluid in all directions. Pressure is $\frac{\text{Force}}{\text{Area}}$, so the same pressure will result in a larger force for a larger area. Hydraulic lifts take advantage of Pascal's Principle by letting you push down on a small area of fluid, while a larger area of that same fluid is used to lift something heavy, like a car. Because the pressure you create is constant throughout the fluid, a small input force produces a large output force. Unfortunately however, there is no free energy to be gained, as the volume of fluid you displace is the same volume that lifts the car. You need to push over a long distance to lift the car just a little. Of course, that is still better than not being able to lift the car at all. The input work, a small force times a long distance, equals the output work, a large force times a short distance.

When a fluid flows through a pipe, the mass of the fluid going in must equal the mass coming out, and the volume of flow must be the same everywhere along the pipe. If the pipe has a narrow section, the flow has to speed up so that the same amount of fluid continues to move.

$A_1v_1 = A_2v_2$

where A is the cross-sectional area of the pipe, and v is the speed of the fluid flow. This is the **continuity equation**. The same principle also causes a stream of water from a faucet to be narrower at the bottom. Gravity accelerates the water as it falls, and as the velocity increases the cross-sectional area gets smaller.

Bernoulli's Principle says that as a fluid flows faster, pressure decreases. Airplane wings are constructed to force air (a fluid) to move faster over the top of the wing compared to the bottom. The decrease in pressure at the top creates a lifting force.

LAWS OF THERMODYNAMICS

The **First Law of Thermodynamics** says that energy can neither be created nor destroyed. It can only change forms. We can express the first law as:

$\Delta U = Q - W$

Here ΔU is the change in internal energy of a particular system, Q is the heat transfer, and W stands for work. This expression was originally developed when people were studying steam engines, where hot steam does work. As you add heat (Q) to a system, its internal energy, U, goes up. When the system does work, its internal energy goes down. The net change in energy is $Q - W$. In physics, work done by the system is considered positive, while work done on the system would be negative. So, if you do work on a system its internal energy increases – two negatives make a positive.

The "system" we are considering here is usually a gas, preferably an ideal monoatomic gas. In that case "internal energy" would be the kinetic energy of the gas particles. In real life however there are interactions between the particles of a gas, and even inside of them if the gas molecules are made up of more than a single atom.

The work done by the system can be a gas pushing a piston inside a cylinder. As the gas expands, it pushes the piston outward a distance d, increasing the volume by an amount equal to the area of the piston times d. Work is equal to Force times distance, and the force here is supplied by the pressure of the gas. Pressure is defined as force per unit of area, so to get the total force exerted on the piston we need to multiply the pressure by the area of the piston. The work done is equal to pressure times area of piston times d, or pressure times the increase in volume: **Work = $P\Delta V$**.

There are several special, *idealized*, cases of thermodynamic processes.

In an **isothermal process**, the temperature remains constant. Since temperature is determined by the average kinetic energy of the particles of the gas, the internal energy does not change. This means that any heat energy added to the system is converted to work (or the other way around). $\Delta U = 0$ so $Q = W$. The heat has to be added very slowly so that it can be converted to work without actually heating up the system. An example would be the evaporation of a small spill of water on your desk. As the water evaporates it expands, pushing against the surrounding air, so it does work. The water molecules remaining in the puddle have lower kinetic energy than the ones that escape into the air, but heat is transferred into the puddle from its surroundings. As the spill evaporates, its temperature remains constant, while the water vapor does work by expanding. All of the heat taken in from the environment is transferred into work, very slowly.

An **adiabatic process** takes place without heat transfer. Q = 0, so ΔU = W. The internal energy changes only due to work done by or on the system. No heat enters or leaves the system, so if the internal energy changes the temperature of the system also increases or decreases. This would be possible if the system is extremely well insulated, or if the process takes place so quickly that there is not enough time for heat to be transferred to or from the environment. An example would be the rapid release of gas from a cylinder of compressed Helium into a balloon. The gas cools as it expands very rapidly and does work adiabatically. Temperature equilibrium is restored soon afterwards through heat transfer from the environment.

Note that when a gas expands into a vacuum, it does not do work because there is nothing to push out of the way. In that situation the expansion of the gas does not cause a decrease in its temperature.

In an **isovolumetric (isochoric)** process, the volume remains constant. Since Work = PΔV, no work is done. W = 0, so Q = ΔU. Any heat transferred into the system goes into changing the internal energy. A bomb calorimeter is a rigid, well insulated container. When we run a chemical reaction inside the container, the energy released by the reaction is converted entirely to heat. No energy is lost as work because the container is rigid and the volume is unchanged. By measuring the increase in temperature, we can know precisely how much energy was produced by the reaction.

An **isobaric process** takes place at constant pressure. When heat is transferred into the system the volume is allowed to expand so that there is no change in pressure. Work is done (the volume changes), and typically the system also heats up so there is a change in its internal energy. Hot gas pushing on a piston in a cylinder would be isobaric if the piston is very easy to move and no pressure builds up.

The **Second Law of Thermodynamics** says that the total **Entropy** of the universe is always increasing. Entropy is disorder or randomness. Any spontaneous process must increase the entropy of the universe.

HEAT ENGINES

You can easily tell the difference between a modern train and a steam train, because a steam locomotive always emits an obvious plume of steam. Although a lot of the heat provided to the steam train's engine does work, a significant quantity escapes into the environment as hot steam. To run a steam locomotive, you must burn fuel to heat up water in the boiler. The boiler is the hot reservoir. The amount of work done is equal to the difference between the heat input to the hot reservoir and the waste heat released to the environment (the cold reservoir):

$W_{net} = Q_h - Q_c$

Here Q_h is the heat input to the hot reservoir, which is supplied by the burning fuel, and Q_c is the heat transferred into the cold reservoir. When there is more of a temperature difference between the two reservoirs, more work can be done.

Any machine that uses heat transfer to do work is called a heat engine. Most of these machines use a **cyclic process**, which means that the properties of the system at the end of a cycle are identical to the properties at the start of the cycle. There is no net change in the internal energy for each complete cycle. $\Delta U = Q - W$, and since ΔU is zero, $W = Q$ or $W_{net} = Q_h - Q_c$.

As we saw in the section on machines, the efficiency of a simple machine is $\frac{\text{Output Work}}{\text{Input Work}}$. In the case of heat engines all of the input work is in the form of heat, while the output work is $Q_h - Q_c$:

Efficiency $= \frac{\text{Output Work}}{\text{Input Heat}} = \frac{Q_h - Q_c}{Q_h}$

That may also be written as Efficiency $= 1 - \frac{Q_c}{Q_h}$. Q_c is the heat transferred to the cold reservoir, which is usually the environment.

The Second Law of Thermodynamics says that Q_c can never be zero in a cyclic process. Work can be completely converted into heat, but heat can never be completely converted into work. So, unfortunately, the efficiency of a cyclic heat engine is never 100%.

A nuclear power plant is a type of heat engine. A nuclear reaction generates heat, which is used to turn water into steam that can do work. Then the steam is allowed to condense back into water to complete the cycle. The condensation releases heat into the environment. You may think that the temperature difference here is limited to the difference between the reservoir of boiling water (100^{o}C) and the ambient environmental temperature, but we can get more work output by increasing this difference. The steam produced by the reactor can be heated further to become superheated steam, and the heat can be discharged into both the air and a nearby body of cool water. The steam that is used to do work recirculates in a closed loop, and the means to cool it back into water is provided by a separate water cooling system. Steam that billows up from the large towers of a nuclear power plant is not part of the steam that does the work. It is simply evaporation of the water that was used for cooling, and it is not radioactive.

A refrigerator is a heat engine that works in reverse. Instead of using a temperature differential to generate work, it does work to create the temperature differential that keeps your food cold.

DICTIONARY

Angular Displacement: The change in an object's angle as it rotates.

Angular Momentum: The momentum of an object rotating about a fixed axis. Moment of Inertia · angular velocity = mass · radius · velocity

Apparent Weight: The force an object experiences as a result of all the forces acting on it. If an object is partially immersed in a liquid its apparent weight is lower than its actual weight due to the upward force exerted by the liquid.

Center of Mass: The weighted average position of all of the mass in a particular system.

Closed System: A system which does not gain or lose mass.

Conservative Force: A force that does an amount of work that is independent of the path taken. Gravity does the same amount of work whether an object falls or if it rolls down a ramp of the same height. Friction on the other hand does more work when the path is longer.

Damping: The tendency of a vibrating object to lose its energy over time.

Drag Force: The force exerted by a fluid on an object moving through that fluid, or the force exerted by air. As an object falls, the drag force increases as the object's velocity increases. Eventually there is no net force on the object so no further acceleration occurs. The object has reached terminal velocity.

Dynamics: Study of forces that affect the motion of objects and systems.

Efficiency of a Machine: $\frac{\text{Output Work}}{\text{Input Work}}$ x 100%

Effort Force: The force exerted by a person on a machine (input force).

Elastic Collision: A collision in which the total kinetic energy is conserved.

Energy: The ability of an object to produce change in the environment or itself.

Equilibrant: The force that puts an object in equilibrium.

Force: A push or a pull.

Free-Body Diagram: A diagram that shows the magnitude and direction of all of the external forces acting on a single object in a given situation.

Free-Fall: Ideal motion influenced by gravity alone (no air resistance).

g: The constant acceleration caused by Earth's gravity. The value of g is approximately 9.81 m/s^2 near the surface of the earth.

Gravitational Mass: A measure of how strongly a body is affected by Gravity. A balance is used to compare the effects of gravity on a known mass to the effects of gravity of the unknown mass.

Gravitational Potential Energy: Energy contained in an object due to its position. This can be converted to kinetic energy. An arbitrary reference level is used to determine the 0 position.

Harmonic Motion: A back and forth type of motion in which the restoring force is directly proportional to the displacement.

Impulse: The product of the average force on an object and the time interval over which it acts.

Impulse-Momentum Theorem: The impulse on an object is equal to the object's final momentum minus the object's initial momentum

Inelastic Collision: A collision in which kinetic energy is lost.

Inertia: The resistance of an object to any change in its motion.

Inertial Mass: The property of a body that determines how much it accelerates as the result of the application of any force.

Interaction Pair: Two forces that are equal and opposite in magnitude.

Isolated System: A system that is free from the influence of a net external force that alters the momentum of the system. Friction is considered an external force.

Joule: A unit of energy/work. 1 Joule = 1 Newton – meter (a force of 1 N applied over a distance of 1 m).

Kinematics: The study of motion – describes the way objects move.

Kinetic Energy: The energy of motion. Kinetic energy = $\frac{1}{2} mv^2$

Kinetic Friction: A frictional force that acts while the object is moving.

Law of Conservation of Energy: In a closed, isolated system, energy is conserved.

Linear Momentum: Momentum, the product of mass and velocity.

Mechanical Advantage: The ratio of resistance force to effort force for a machine.

Mechanical Energy: The sum of potential energy and kinetic energy.

Moment of Inertia (Rotational Inertia): Resistance to torque. $I = mr^2$

Momentum: Mass times velocity.

Newton: A unit of force. One Newton is the amount of force that will give a 1 kg mass an acceleration of 1 m/sec^2. 1 N = 0.225 lb.

Normal Force: A perpendicular contact force exerted by a surface on a mass.

Power: Work done per unit of time. Measured in Watts

Projectile: Any moving object that moves only under the force of gravity after initial thrust.

Resistance Force: The force exerted by a machine (output force).

Resultant: a vector that is the sum of two or more other vectors.

Rigid Rotating Object: A mass that rotates around its own axis.

Rotational Equilibrium: Net torque is zero. There is no change in angular momentum. The object may be rotating or accelerating in a straight line.

Rotational Inertia: Resistance to torque. $I = mr^2$

Simple Harmonic Motion: Harmonic motion with no energy loss (no friction) to slow the motion down. The amplitude remains constant.

Static Equilibrium: This exists when there is both rotational equilibrium and translational equilibrium.

Static Friction: A force that prevents an object from moving until a certain minimum force is applied. Static friction opposes the applied force, and increases until the object begins to move.

Tension: A force transmitted by a rope (or string or chain) that is pulled tight by forces acting on both ends.

Torque: A measure of how effectively a force causes rotation. Its magnitude is the force times the length of the lever arm.

Translational Equilibrium: The acceleration of the center of mass is zero, although the object may be rotating and or experiencing a change in angular momentum. The net force is zero.

Translational Momentum: (Linear) momentum.

Vector Resolution: The process of breaking down a vector into its components.

Watt: 1 Watt = 1 joule/sec

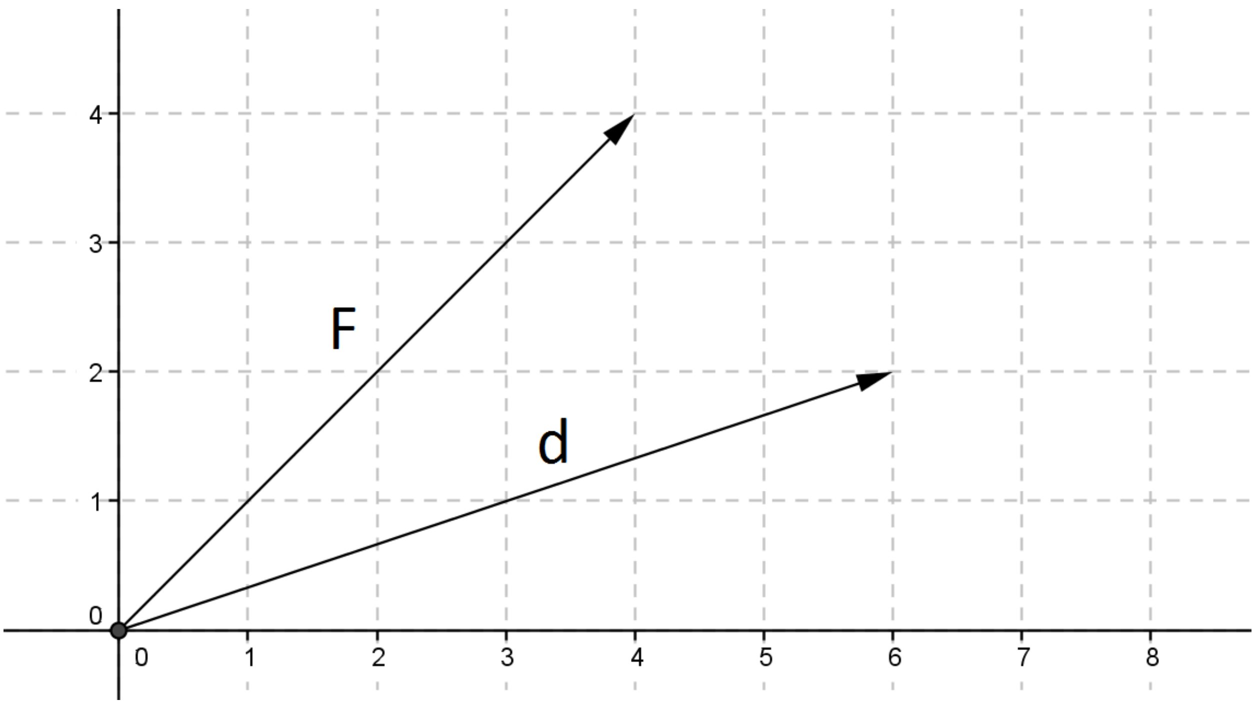
Work: The transfer of energy by mechanical means. Calculated as Force times distance.

Work-Energy Theorem: The work done on an object is equal to the change in its kinetic energy.

EXTRA STUFF

Vectors and Work: The dot product

The work is $|\vec{F}|$ cos θ multiplied by the displacement, $|\vec{d}|$, where θ is the angle between the force vector and the displacement vector. This is the dot product of $\vec{F}$ and $\vec{d}$. The dot product is a scalar. If α is the angle of $\vec{F}$ with a horizontal axis, and β is the angle of $\vec{d}$ with that same axis, then cos θ = cos (α – β). Trigonometry tells us that cos (α – β) = cos α cos β + sin α sin β.



$\vec{F}$ has x-component F_x and y-component F_y. $\vec{F} = (F_x, F_y)$. $\vec{d}$ has x-component d_x and y-component d_y. $\vec{d} = (d_x, d_y)$.

Therefore,

cos θ = cos α cos β + sin α sin β

In this case cos α is $\frac{F_x}{|\vec{F}|}$ and cos β is $\frac{d_x}{|\vec{d}|}$, while sin α is $\frac{F_y}{|\vec{F}|}$ and sin β is $\frac{d_y}{|\vec{d}|}$:

$$\cos\theta = \frac{F_x}{|\vec{F}|}\cdot\frac{d_x}{|\vec{d}|} + \frac{F_y}{|\vec{F}|}\cdot\frac{d_y}{|\vec{d}|} = \frac{(F_x\cdot d_x + F_y\cdot d_y)}{|\vec{F}||\vec{d}|}$$

$\left|\vec{F}\right| \left|\vec{d}\right| \cos \theta$, the dot product of $\vec{F}$ and $\vec{d}$, is $F_x d_x + F_y d_y$.

Torque: The Cross-Product

In physics, the cross product of two vectors is used to represent torque. The cross product of two vectors $\vec{d}$ and $\vec{F}$, $\vec{d} \times \vec{F}$, is a vector with length $\left|\vec{d}\right| \left|\vec{F}\right| \sin \theta$. This vector is perpendicular to both $\vec{d} \times \vec{F}$, so the cross product only has meaning in three dimensions. The magnitude of the cross-product vector is the area of the parallelogram determined by $\vec{d}$ and $\vec{F}$. We use the right-hand rule to determine the direction. Use your fingers to push $\vec{d}$ into $\vec{F}$, and take the direction of the cross product vector to be where your thumb points. To find the cross-product you must take the determinant of $\begin{matrix} i & j & k \\ F_1 & F_2 & F_3 \\ d_1 & d_2 & d_3 \end{matrix}$

The vector is (i $\begin{vmatrix} F_2 & F_3 \\ d_2 & d_3 \end{vmatrix}$, - j $\begin{vmatrix} F_1 & F_3 \\ d_1 & d_3 \end{vmatrix}$, k $\begin{vmatrix} F_1 & F_2 \\ d_1 & d_2 \end{vmatrix}$)

Electronics Quiz

1. To measure a resistor:
    a. Set the meter to OHMS
    b. Make sure the resistor is connected to a source of voltage
    c. Set the meter to DC VOLTS
    d. Just look at the color coding on the resistor; measurement is never needed

2. Current is measured:
    a. Only in milliamps, because Amps are too dangerous
    b. In Ohms
    c. By inserting the meter into a working circuit
    d. In Volts

3. To measure the voltage across a resistor:
    a. Connect the meter in series with the resistor
    b. Connect the meter in parallel with the resistor
    c. Set the meter to OHMS
    d. Set the meter to AMPS

4. Two 10K Ohm resistors are connected to a voltage source in series. The total resistance is:
    a. 5K Ohms
    b. Impossible to determine because it varies with the voltage
    c. 20KOhms
    d. Impossible to determine because it depends on the current

5. Next, these same resistors are connected to the same voltage source in parallel. Now:
    a. More current flows in the circuit because there are two paths for it to take
    b. The voltage goes down because the resistance is less
    c. Less current flows in the circuit because it has to fill up two paths
    d. The voltage goes up because the resistance is less

6. Two 100 Ohm resistors are connected in parallel. The total resistance is:
    a. 200 Ohms
    b. 1/100 Ohms
    c. 100 Ohms
    d. 50 Ohms

7. A good material for making semiconductors like diodes and transistors is:
    a. Carbon
    b. Copper
    c. Silicon
    d. Electrons

8. Light emitting diodes (LED's) produce light because:
    a. Current causes them to heat up and give off light
    b. When the electrons are blocked from flowing in the wrong direction their energy is converted to light energy
    c. When electrons drop into "holes" they give of energy in the form of photons
    d. When electrons move they leave behind "holes" that produce photons

9. In P-type doping of silicon:
    a. Holes are added to the silicon by removing electrons from it, producing P-type material
    b. A substance with five outer electrons is added to create "holes"
    c. A substance with three outer electrons is added to create "holes"
    d. Some of the silicon atoms are removed to create "holes"

10. Transistors are useful because they:
    a. Can act as capacitors, so they can store voltage
    b. Don't need doping
    c. Are semiconductors
    d. Act as little switches that can block current or allow it to flow

True or False:

1. Capacitors can be used in a circuit instead of batteries because they store voltage

____

2. A diode is used in a circuit to keep current from flowing in the wrong direction

____

3. Some screwdrivers contain a diode to protect you in case you accidentally touch an electrical wire

____

4. A longer wire has more resistance than a shorter wire of the same diameter

____

5. According to Ohm's law, current increases as resistance increases

____

Electronics Quiz II

1) A 100 Ohm resistor and a 470 Ohm resistor are connected in series, to a 3V source.
    a. What is the total resistance?
    b. What is the current in this circuit?

2) The same two resistors are now connected in parallel
    a. What is the total resistance
    b. What is the total current in this circuit

3) Why might you see a small light bulb light up when checking its resistance?

4) How could you tell if this same light bulb was burned out using only your meter?

5) Five 3V light bulbs are connected to a 3 Volt source. Should they be connected in series or in parallel to show their maximum brightness?

6) Which substance conducts current better, graphite or diamond? Why?

7) In our orange battery, why do electrons flow from the zinc plated screw to the copper wire?

8) What would make a better battery, a tomato or a doughnut? Why?

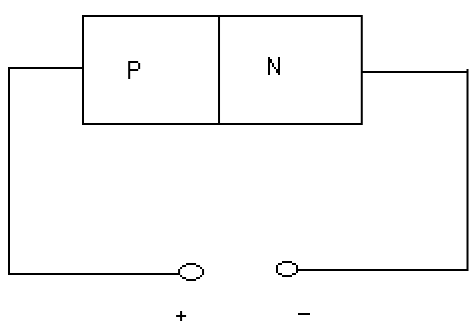
9) If you place a strip of copper in a silver nitrate solution, silvery stuff starts appearing on the copper.
    a. Why does this happen?
    b. Why does the solution turn blue after a while?

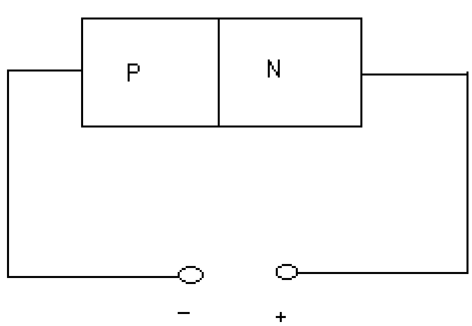
10) What conducts electricity better, distilled water or a salt solution? Why?

11) When you measure the resistance of a salt solution, it is low at first, but then it quickly increases and stays steady. What happened?

12) When we passed current through a salt solution, bubbles formed at one of the electrodes. What caused these bubbles, and did they form on the positive or negative electrode?

13) Which diode will have current flowing across it? Place a check mark next to the picture.





14) How does a LED produce light?

15) If you need a 9V battery, but have only 1.5V ones, should you connect them in series or parallel?

16) We used a zinc coated nail for our orange battery. Why did it look different afterwards?

17) Why would someone think of using air at one terminal of a battery? Does that work?

18) Can you use different metals or compound to create a single battery that produces 9V? Why or why not?

19) A circuit has a 100 Ohm and a 1K Ohm resistor connected in parallel. The voltage is 9V. What is the voltage drop across each resistor?

20) How do you make a battery using coins?